EARTH'S CIRCUMFERENCE[1] (for classes 9-10)

Purpose: *Learn to use circles and triangles for global measurement/mapping. Learn about Eratosthenes, Al-Biruni, Al-Mamun, Aryabhata, Al-Idrisi and their times. See how celestial events influence culture. And how geometry/knowledge progressed along with rational thinking/cultural exchange*

Topics Covered:

1. Geometry: Circumference, Angles in a Circle, Days in a Year, Parallel lines and Angles, Right Angled Triangles. Trignometric functions: Sin, Cosine, Tan

2. Geography: Latitude, Longitude, Tropics, Axial Tilt, Solstices, Equinoxes and Local Festivals associated with them: *Nauroz, Lohri, Vaisakhi*; World Map of Al-Idrisi

3. History: Eratosthenes of Alexandria, Library of Alexandria, Greek Rationalism, Arabs and Europe, Alexander, Selucus and Ptolemy, Ambhi and Raja Poru. Golden Age of India: Art and Sciences in the Gupta Empire, *Shakuntila*, *Panchtantra*. Al-Mamun and *Bayt Al-Hikmah*.

Background Material: *Poems in Seraiki for children* on (i) Eratosthenes and Circles; (ii) Al-Biruni and Triangles; (iii) Angles, Triangles; (iv) Circle; (v) Galileo and Earth's Rotation; (vi) Pythagoras Theorem
Zoya Science School Diyan Nazman, Nawab Muztar 2015

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Q.Mushtaq and A.L.Tan: *Mathematics: The Islamic Legacy*, Quaid-e-Azam University, 1990.
Great Ages of Man—*Age of Enlightenment*, Peter Gray and *Classical Greece* C.M.Bowra, Time-Life Books, Amsterdam, 1991
Images from Wikipedia; Urdu translation by Obaid Bhutta, Tahir Kursheed

[1] Based on work (lectures and measurements) by Sarmad Khawaja during April-September 2015 with students of Zoya Science School Nala Khudadad, Basti Dari Wala, Union Council Alurid, District Muzaffargarh.

Earth's Circumference—Method of Eratosthenes

Geometry (*geo*—earth; *metry*—measurement): Eratosthenes (276—194 BC) first calculated the Earth's circumference using properties of circles in two steps based on observing the Sun simultaneously at two places: at Aswan (which lies on the Tropic of Cancer and so on June 22 the Sun makes a 90° angle with it shining directly on the bottom of a well in Aswan) and at Alexandria.

Step 1: The earth being round as the sun shines directly over Aswan it shines at an angle in Alexandria (which is several hundred kilometers north of Aswan). Eratosthenes measured the angle made by the sun at Alexandria on June 22 (at noon) by finding Tan^{-1} of the ratio of the lengths of a tower in Alexandria and its shadow, which is the same as the angle of Aswan and Alexandria from the Earth's center ($\angle$COD=7°12' or 1/50 of 360°)[2];

Step 2: He measured the distance between Aswan and Alexandria (800 km) (from number of steps a camel took to go between the two cities multiplied by length of the camel's step).
From the equality of ratios:

$$\rightarrow \quad \frac{800}{\text{Earth's Circumference}} = \frac{7°12'}{360°} = \frac{1}{50}$$

Earth's circumference is 50 x 800 = 40,000 km

Finding Angle $\angle$COD

Sun's rays AB and OD are parallel.
$\therefore \angle$COD = $\angle$ABC

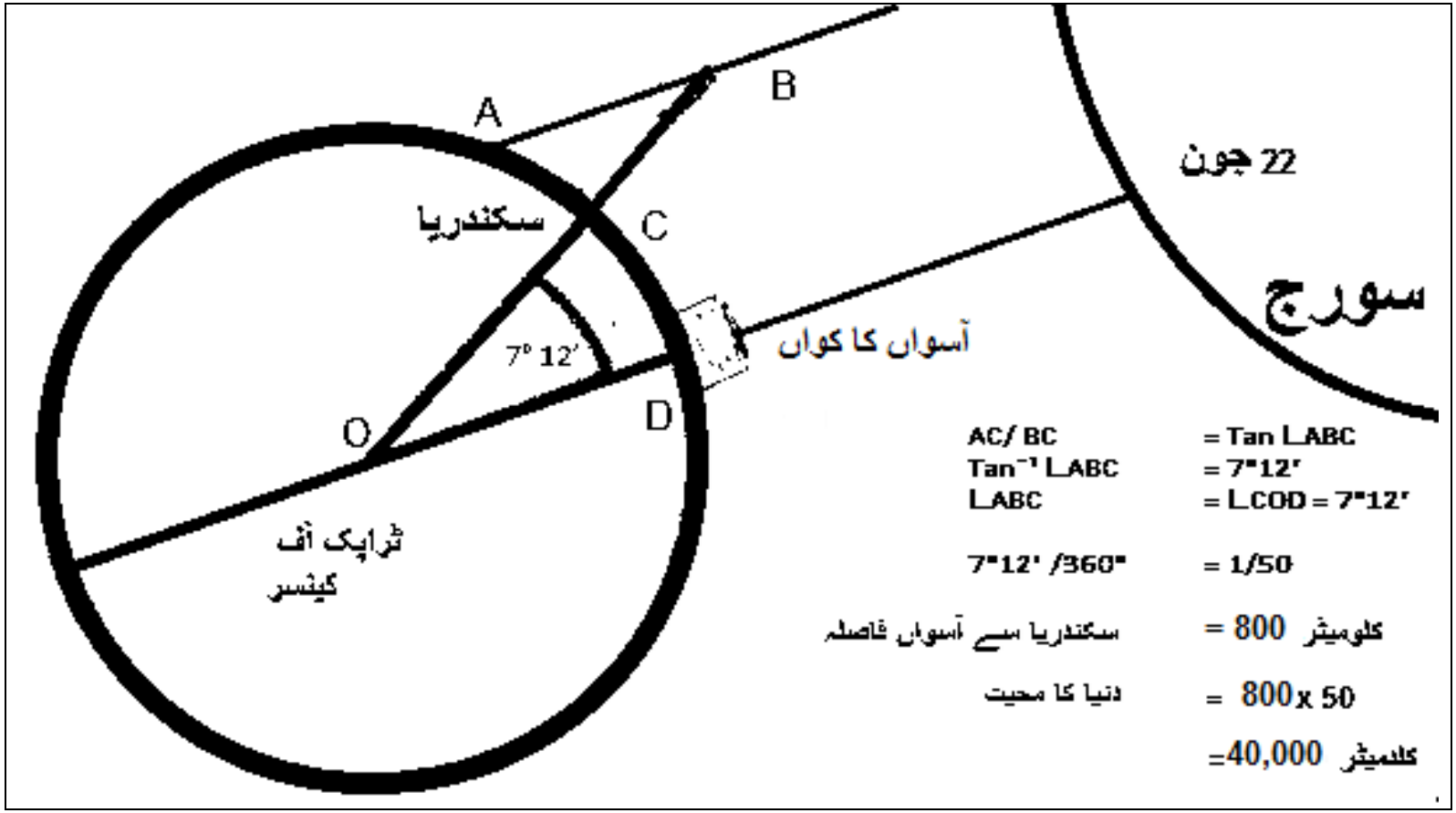
In Δ ABC made by the sun's ray AB, the tower BC and the tower's shadow at noon AC the ratio of the perpendicular to the base

$$= \frac{AC}{BC} = \text{Tan} \angle ABC$$

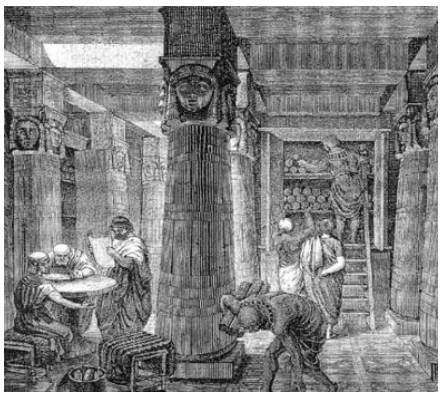
$\therefore \angle$ABC = Tan^{-1} $\angle$ABC = 7° 12' = $\angle$COD

[2] Adding $\angle$COD to the latitude of the tropic of Cancer gives the latitude of Alexandria

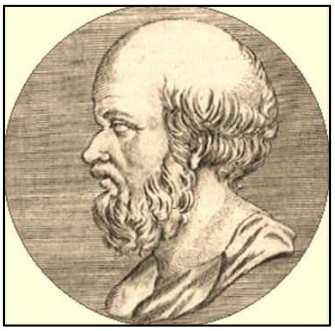
Eratosthenes Method to Measure Earth's Circumference



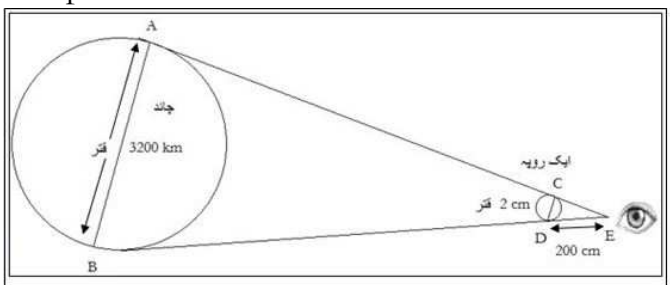
Eratosthenes (276—194 BC) born in Libya was head of the **Library of Alexandria** the highest official appointed by the king and tutor of the king's son. The Library was part of the *Musaeum* where over 1000 scholars researched, lectured, collected and translated documents from India, Greece and Iran, including Euclid—founder of Geometry (around 450—350 BC), Archimedes (287—212 BC)—founder of engineering, Aristarchus (310—230 BC), who first proposed the heliocentric system of the universe. The *Musaeum* was built by Ptolemy, general of Alexander and after him ruler of Egypt, though it was Alexander's idea to collect books from his empire into one place.



Musaeum of Alexandria (place of the Muses—dedicated to nine Greek goddesses (muses), daughters of Zeus, personifying arts, literature and music)



Eratosthenes



$$\frac{BE}{DE} = \frac{AB}{CD}$$

$$BE = \frac{AB}{CD} \times DE = \frac{3200}{2} * 200 = 320000\ km$$

Eratosthenes described the earth in his book *Geographica* bringing into it reports of Alexander's surveyors, explorers and voyagers. He mentions China but did not account for North Europe, North Asia, South Africa.

The estimate of the Earth's circumference (and its diameter) enables calculating the moon's diameter—by observing, during a lunar eclipse, that the moon takes a fourth of time to eclipse as compared with the time it takes to come out from the earth's shadow. Its diameter is thus a fourth of the earth's or 3200 km ($= \frac{40000}{4\pi}$); And the distance to the moon may be calculated using a one Rupee coin (diameter 2 cm). It hides a full moon when placed 200 cm from the eye. The distance of the moon from earth is therefore 100 times more than the moon's diameter or 320000 km.

Greece and India: Until Alexander's (356—323 BC) invasion of India in 326 BC Iranians ruled over the territory west of Indus. They perpetually fought the Greeks. Xerxes' (486—465 BC) army had soldiers from his vast empire: Indians wearing cotton dresses, Afghans, Libyans, Arabians, Egyptians, Turks, South Europeans. They subdued Greece and raided Athens in 480 BC. A similar immense but disorderly polyglot army under Darius 130 years later was beaten by Greek phalanxes and routed by swift dashes of Alexander's cavalry who then declared that Greeks were now completely free from Iranian rule.


Battle on the Jhelum

Alexander set up new cities in Kandahar (Afghanistan) and Khojant (Tajikistan) where he married Roxana (who bore him a child after his death). He was joined by Raja Ambhi of Taxila who ruled the doab (area between 2 rivers) between Indus and Jhelum to defeat Raja Poru (326 BC) who ruled the Jhelum—Ravi doab. Alexander made Poru his satrap like so many kings he defeated.

In the battle on the east bank of Jhelum 1000 mounted archers first harassed Poru's cavalry, which was then attacked and decimated by Alexander's battle-hardened cavalry. As the Greek phalanxes advanced the cavalry encircled the battlefield. The Indian infantry crushed in the center confusing the elephants, who trampled them, while the Greek infantry and cavalry killed them.


Greek coin showing an Indian elephant on the run before a Greek horseman

Alexander's army returned home via Indus and Arabian Sea and through Baluchistan. Ten thousand died crossing the Baluch desert. Alexander died in Babylon (Iraq)—the greatest conqueror of the World until Chengiz Khan[3].

Alexander's empire split into three after his death. Most of Asia went to Selucus who gave to Chandragupta Maurya the west bank of Indus upto

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[3] کتھے ہے سلطان سکندر موت نہ چھڈے پیر پیغمبر
کرلے آج کرنی دا ویرا مڑ نہ ہوسی آون تیرا — hahS helluB

Kandahar in exchange for peace and 500 elephants. In his book *Indica*, Megasthenes—Selucus' emissary at Chandragupta's court in Pataliputra (now Patna, 1000 km west of Delhi and 300 km south of Khatmandu) estimated the North-South length of India at 4000 km or about a third more than the actual length of 2900 km, which a generation later Eratosthenes adjusted downward by 500 km.

In Afghanistan/KP the Greeks ruled another 300 years followed by the Kushan Empire of central Asians under whose greatest king Kanishka Peshawar and Taxila became centers of learning and Buddhism. Meanwhile, in central and Northern India Chandragupta Maurya erased all trace of Alexander's authority and started the Mauryan dynasty, which ruled India and Afghanistan for 137 years, including Chandragupta's grandson Asoka. Ashoka's empire crumbled after him, but as Will Durant says he "accomplished one of the greatest tasks in history. Within two hundred years after his death Buddhism had spread throughout India."

The Golden Age of India came in the Gupta Empire (320—550).

Greek Rationalism and Islam: Alexander and Ptolemy studied with Aristotle (384—322 BC) a student of Plato (427—347 BC) who studied with Socrates (470—399 BC). The Greeks used logic and reason to address matters of belief and practice—a way of thinking called *Rationalism*.

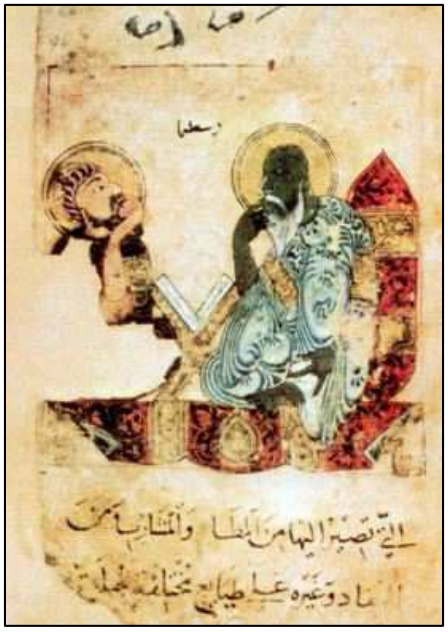
Socrates was sentenced to death for corrupting the minds of the youth of Athens. In the picture he drinks the poison while his students mourn. "The gods are an invention of clever statesmen who use them as night watchmen to frighten men into decency"

The Socratic Method (or scientific method or dialectics or logic or rational method) examines ideas by questioning them. They are rejected if found to self-contradict, and replaced with other ideas to be similarly questioned. Science needs such thinking. It demands observation, patient collection of facts, experimentation and boldness in framing ideas, an independent and inquisitive spirit, an ability to see things as they are.

Muslim scholars called Aristotle *Al-Muallim-Al-Awwal* (First Teacher). Al-Farabi (872—950 AD), Ibn-Sina (980—1037 AD) and Al-Biruni (973—1048 AD) debated Aristotle. Influenced by Aristotle caliph Al-Mamun (786—833) and the Mutazili sought to reconcile the

justice of an all-powerful God with evil in the world. The Ulema and later Ibn-Hanbal opposed them saying: *everything a believer needs to know about faith and practice is in the Quran and Hadith*. Ibn-Hanbal suffered from the Abbasids' *mihna*—investigation and punishment of Ulema opposing the Mutazili understanding of the created nature of Quran. Ibn-Hanbal rejected religious rulings (*Ijtihad*) of jurists and *bida* (innovation) as speculative theology (*Kalam*).

Later Ibn-Gazali (1058—1111) gave a further blow to Muslim rationalism with his book *Tahafut al-Falasifa (Incoherence of the Philosophers)*, which denounced Al-Farabi and Ibn-Sina. Ibn Rushd (1126—1198)(called Averroes in the west) wrote *Tahafat al-Tahafat* (*Incoherence of the Incoherence*) to defend using Aristotle in Islamic thought. He said: natural phenomenon follow laws that God created. But al-Ghazali said that whatever happens is because God wills it. Ibn Rushd's book was not well-received by Muslim scholars. Muslims fell into the stranglehold of Mullahs and remain in their fold. Argument from authority, indifference to facts, blind faith darken the mind of Muslims. The driving force of the scientific-technological revolution is creativity. Creativity is the essence of *Ijtihad*. Ibn-Hanbal and Ibn-Ghazali closed the door of *ijtihad* to the Muslims! (*Here: Iqbals' verse "For 300 years the doors of the tavern are closed"*).



Aristotle: Al-Muallim-Al-Awwal in Arab painting

Ibn-Rushd: physician, mathematician, philosopher popularizer of Aristotle

Meanwhile the West became Averroists—followers of Ibn-Rushd—adopting him as the founding father of rational thought.

The Hanbali school of Islamic law is dominant in Saudi Arabia and Qatar who export these ideas to other countries. The Salafi movement is inspired by Hanbali sharia and strictly follows *al-salaf al-salih* (pious forefathers).

Earth's Circumference: Calculated by Students of Zoya Science School Nala Khudadad

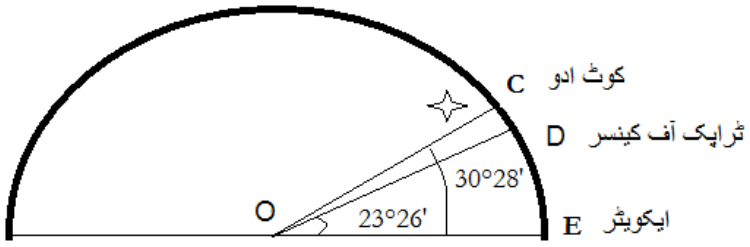
First: Using the method of Eratosthenes the students found the angle of Kot Addu and Tropic of Cancer from the Earth's center (∟COD = 7°) on June 22 when the sun was at 90° above Tropic of Cancer from the angle made by the Sun and the shadow of a stick (they checked this result from the angle obtained by subtracting latitude of Tropic of Cancer from latitude of Kot Addu (30°28' - 23°26' = 7°2')

And, following Al-Mamun (see next section) they found that 1°N latitude covers (approximately) 107.28 km: distance between Mianwali (32°59' N 71°54' E) and Kot Addu (30°28' N 70°57' E) = 270 km; Difference in latitude is (32°59' - 30°28') → 1°N = 270 km ÷ (32°59' - 30°28').

∴ distance between Kot Addu and Tropic of Cancer is 754.6 km [(32°59' - 23°26')*107.28]. From the equality of ratios:

$$\rightarrow \qquad \frac{754.6}{Earth's\ Circumference} = \frac{7°}{360°} = \frac{1}{51.4}$$

Earth's circumference is 51.4 x 780 = 38,806 km



∟COD = ∟COE - ∟DOE

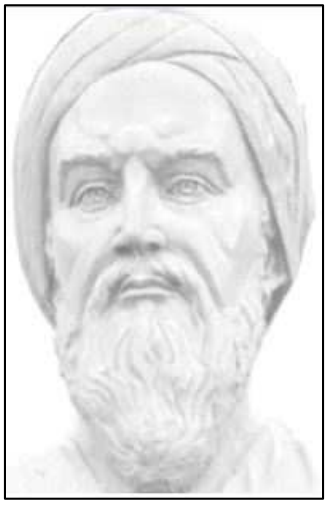
= 30°28' - 23°26'

∟COD = 7°2'

∟COE = لیٹیٹیوڈ کوٹ آدو = 30°28'

∟DOE = لیٹیٹیوڈ ٹراپک آف کینسر = 23°26'

Earth's Circumference: Other Measurements



Abu Rayhan Al-Biruni (973-1048). *Nisbat* Al-Biruni means "from outer place" i.e. from Khwarezm (presently Uzbekistan) away from Iran. Instead of observing the Sun simultaneously from two places and walking across deserts (as done by Eratosthenes and Al-Mamun) to estimate the earth's circumference Al-Biruni measured it from a single location using properties of triangles and making two calculations: the height of a hill near Pind Dadan Khan (derived from the angles ∟A and ∟B the hill top makes with two points on the plain and the distance 'd' between the two points); and the angle the hill top makes with the horizon. Al-Biruni estimated the Earth's radius to be 6340 km (and circumference 39838 km). A similarly accurate estimate was calculated in the West only in 16th century. At age 17 Biruni calculated the latitude of Kath (capital of Khwarazm) using the maximum altitude of the Sun. Similarly, the latitude of Kot Addu = 30°--the angle made by the Sun on June 22 i.e. 7° plus 23° (the latitude of Tropic of Cancer where the Sun shines at 90° on this day).

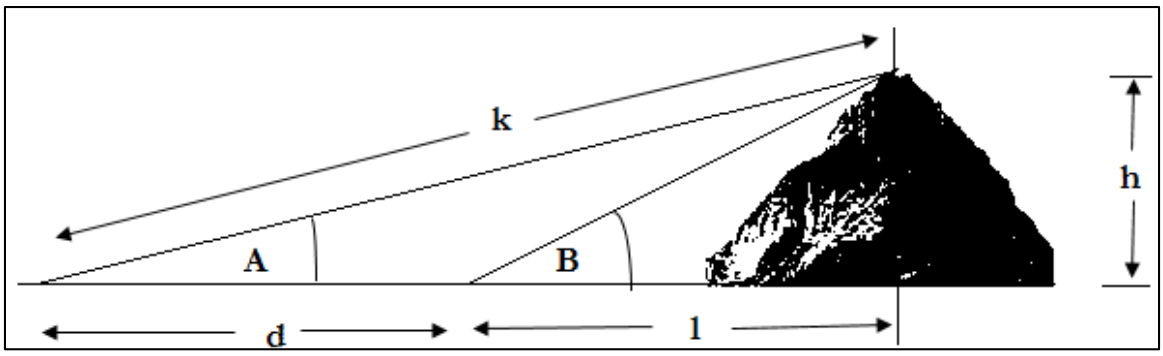
His book (one of 103) *Kanoon Masudi* (encyclopedia of geography and astronomy) gives the coordinates (latitudes and longitudes) of six hundred places, which he visited. In 1017 and later he travelled with Mehmud Gazni to India and wrote *Tarikh Al-Hind* in 1030 which studies Hinduism and its relationship with the Greeks. He studied religions and saw different cultures as related and interlinked because he said: *they are made by humans who are all related.*

How Al-Biruni[4] **calculated the height of the hill** or expressing 'h' in terms of ∟A, ∟B and 'd' which are easily measured.

$$\frac{h}{d+l} = \tan A$$

$$\therefore\ h = d\tan A + l\tan A \ \ldots\ldots\ldots\ldots (1)$$

[4] Al-Biruni was as well geographer, Mathematician, Scientist, author of *Tarikh-al-Hind*. He studied World religions not to prove them wrong but to establish the common human element in them so as to connect them and their cultures.



$$\frac{h}{l} = \tan B$$

$$\therefore\ l = \frac{h}{\tan B}$$

Inserting value of l in equation (1) gives

$$h = d\tan A + \frac{h}{\tan B}\tan A$$

$$h - \frac{h\tan A}{\tan B} = d\tan A = \frac{h}{\tan B}(\tan B - \tan A)$$

$$\frac{h}{\tan B}(\tan B - \tan A) = d\tan A$$

$$\therefore h = \frac{d\tan A\tan B}{\tan B - \tan A}$$

Calculating radius 'r' of Earth and circumference when ∟DAB = ∟a and 'h' the height of the hill are known. Also known that ∟DAC = 90° and since line AB is tangent to the Earth's horizon at B ∟ABC = 90°

$$\sin\angle BAC = \sin(90 - a) = \cos a = \frac{r}{r + h}$$

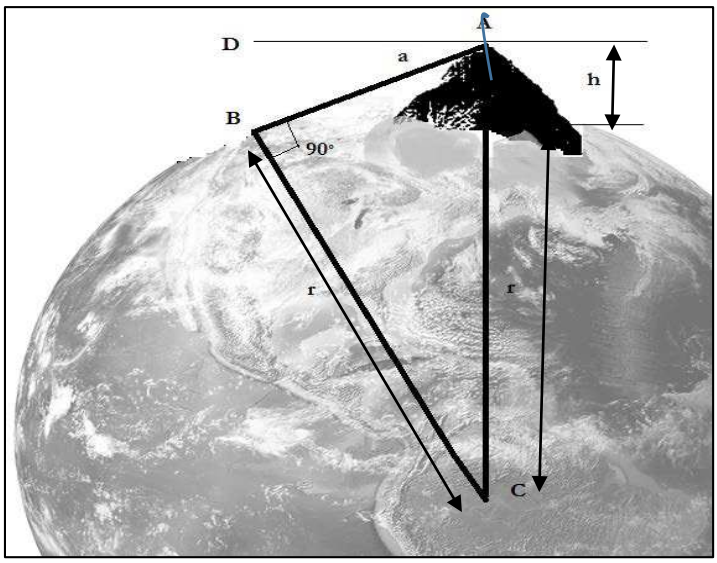
$$\therefore r\cos a + h\cos a = r$$

$$r - r\cos a = h\cos a$$

$$r(1 - \cos a) = h\cos a$$

$$\therefore r = \frac{h\cos a}{(1 - \cos a)}$$

and Earth's circumference $= \frac{2\pi h\cos a}{(1 - \cos a)}$



Al-Mamun's (786—833) team of geographers in *Bayt-al-Hikmah* (House of Knowledge) in Baghdad determined how much 1° latitude (34° and 35°N) is in miles (= 111.7 km) and multiplied it by 360 to find the Earth's circumference: To determine this two teams set out North and South in a field near Mosul (in Iraq) until the angle of the Sun changed by 1° (corrected for the Sun's declination) and measured the distance covered. Al-Biruni says in *Kitab Tahdid Al-Amakin* (*Coordinates of Cities*) that Al-Mamun's team knew from Greek books that 1° latitude is 500 *stadia* but it could not determine the length of the *stade* (length of sports stadium) from the translations[5].

If similar teams set out from Kot Addu (30.4°N), the one going North on the Karor-Bhakkar Road would stop in Solhanwala (31.4°N) (road distance 108 km from Kot Addu) and the one going South on Alipur-Jatoi road would stop in Alipur (29.4°N) (road distance 128 km from Kot Addu). The Solhanwala team would calculate the Earth's circumference to be 360 x 108 km = 38880 km; and the Alipur team 360 x 128 km = 46080 km.

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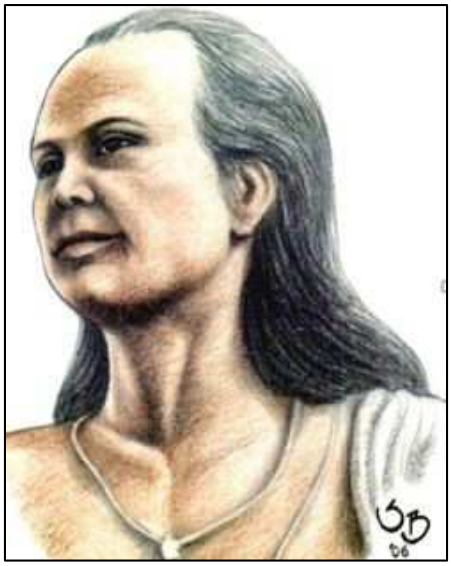
[5] Al-Mamun's *Bayt* was a center of science learning and library with Arab translations of Greek books on philosophy, science, mathematics. It included Ptolemy's *Al-Majisti* (the greatest) (English--*Almagest*) the encyclopedia of Greek Astronomy, Mathematics and Science with description of the motion of stars and planets, and estimates of distances between them. The *Bayt* was destroyed by the Mongols in 1258.

Al-Mamun's scholars knew the mathematical astronomy of **Aryabhata** (476—550) as well about his estimate of the Earth's circumference of 3393 *yogana* (error of 1 percent) from his text *Aryabhatia* which is a summary of Indian mathematics in 118 verses. It covers arithmetic, zero, calculation of π, algebra, plane and spherical trigonometry. As well, quadratic equations, sums of power series and table of sines— preceding Al-Khwarimi's *Al-Jabr wal Muqabila* (considered to be the first book of Algebra) by about 300 years. A lot of Al-Kwarizmi's Algebra came from Indian sources.



India's 1st space satellite (1975-1992) was named Aryabhata

The Earth making a revolution produces a daily rising and setting of the stars and planets. The apparent motion of heavenly bodies is only an illusion just as a passenger in a boat moving downstream sees the stationary (trees on the river banks) as traversing upstream, so does an observer on earth see the fixed stars as moving towards the west at exactly the same speed (at which the earth moves from west to east)— Aryabhata (from his work *Aryabhatia* Section *Gola* 9).

Two lyrical examples of Indian Algebra (from Will Durant): *Out of a swarm of bees one-fifth part settled on a Kadamba blossom; one third on a Silindhra flower; three times the difference of those numbers flew to the bloom of a Kutaja One bee, which remained, hovered about in the air. Tell me, charming woman, the number of bees…… Eight rubies, ten emeralds, and a hundred pearls, which are in thy ear-ring, my beloved, were purchased by me for thee at an equal amount; and the sum of the prices of the three sorts of gems was three less than half a hundred; tell me the price of each, auspicious woman***.**

Golden Age of India. Aryabhata lived in the **Gupta empire** (320—550) during which science and arts flourished: Al-Mansur (753–774) got books on mathematical astronomy and arithmetic by Brahmagupta (598-668) translated into Arabic by Al Fazari called *Sindhind* and *Arakand*. Through them the Arabs adopted Indian numerals and learned the scientific system of astronomy. According to Will Durant "The miscalled 'Arabic' numerals are found on the Rock Edicts of Ashoka (256 BC) a thousand years before their occurrence in Arabic literature." And Simon Laplace (1749—1827) (known as the French

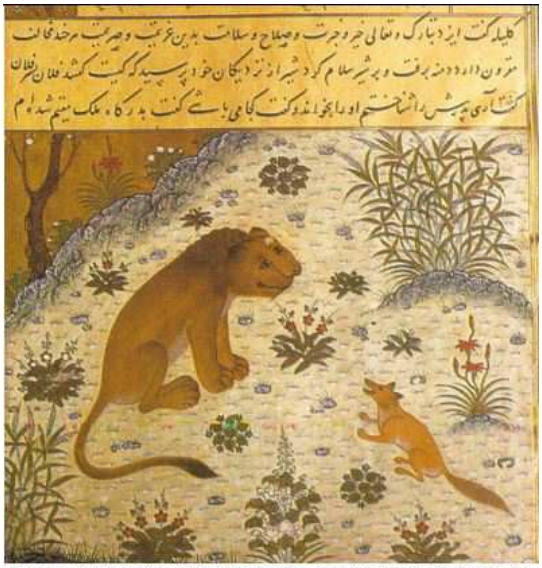
Newton) says: "It is India that gave us the ingenious method of expressing all numbers by ten symbols, each receiving a value of position as well as an absolute value."

Shakuntila

In India's Golden Age also Kalidas wrote *Shakuntila* (protected one) the story of a girl abandoned in a forest and protected by the birds who is found and raised by kind people eventually to marry a king. *Shakuntila* influenced Goethe; it was made into songs, operas and music by western composers (including Franz Schubert 1797—1828 of Austria).

Vishnu Sharma wrote *Panchatantra* (five texts) a collection of stories—one of the most translated story books in history. It was translated into Arabic by Al-Mansur and became best seller in Baghdad (after the Quran). The story tells how Vishnu Sharma taught wisdom to three foolish sons of a king by narrating meaningful stories.

Sushruta Samhita a classical Sanskrit text on medicine was translated in Al-Harun's *Bayat* into Arabic as *Kitab-e-Susrud*. It had 84 chapters covering 1120 illnesses, 700 medicinal plants, and 122 preparations (its explanation of diseases, such as, hypertension, bladder stones, obesity, diabetes matches modern symptoms).



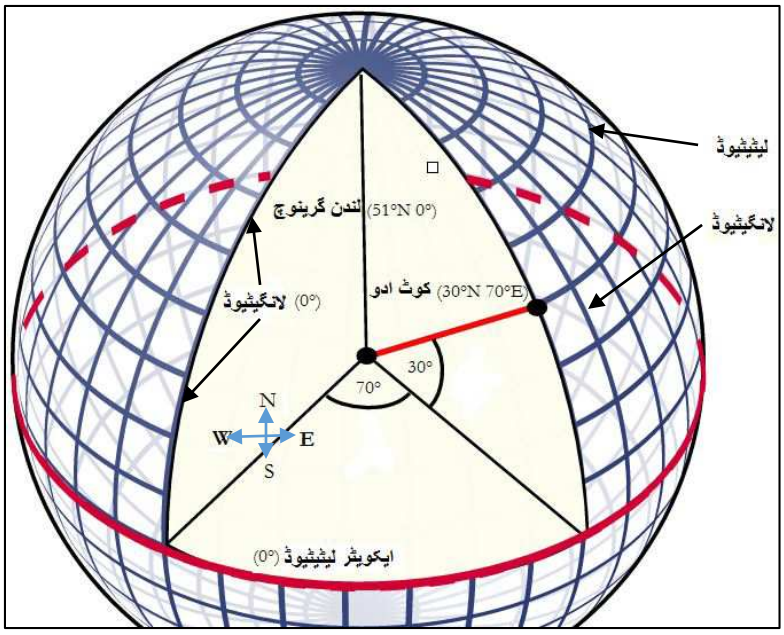
Iranian translation of Panchatantra showing the jackal-Vizir trying to lead his lion-king into war and destruction

Al-Razi's *Al-Hawi fi tib* containing 25 volumes written in 900 which has 'much Indian knowledge' from *Susruta Samhita* was translated into Latin in 1279 for the French King of Sicily and called *Liber Continens.*

Will Durant says: "The great Caliph Haroun-al-Rashid imported Hindu physicians to organize hospitals and medical schools in Baghdad."

Geography: *Geo* (Earth) *graphein* (write, description) is about the processes and patterns in the earth. Eratosthenes first used the word *geographica*.

Mapping the World: his book *Geographica* has maps of places travelled by Greek sailors. Abdullah Ibn Al-Idrisi (1154-1192) an Arab Muslim made the 1st comprehensive world map on which he worked over 15 years with the Christian king Roger II of Sicily. It was published in Arabic and called *Nuzhat Al Mushtaq* and *Tabula Rogeriana* (in Latin). The map was made from first hand observations and older Greek and Iranian maps (which were accurate descriptions of closed seas, such as Mediterranean); as well *Kitab Surat Al-Ard* (Book of the Description of the Earth—a lone surviving copy is in the museum of the University of Strasbourg in France) by Al-Khwarizmi who worked in Al-Mamun's *Bayat Al-Hikmah*. Al-Mamun (786—833) had also commissioned a world map with Al-Khwarizmi. It was published as an encyclopedia and is presently in *Topkapi Serai* (Istanbul museum). It has recognizable outlines of Europe, Asia and Africa as seen by Arab sailors (who by this time had reached the shores of China).



Earth's cross-section showing latitudes and longitudes and how they are determined. Kot Addu (30°N70°E) is 30° North from an (imaginary) line from the Earth's center to the Equator; and 70° East of the Prime Meridian (zero longitude)

Latitude: Al-Idrisi's map has circles parallel to the equator (latitudes) and circles going through North and South poles (longitudes) to specify the positions of places on the Earth. His zero longitude goes through Mecca. Hipparchus (190—120 BC) first specified the positions of places on the Earth using coordinates to which his work on spherical trigonometry led him. He suggested measuring latitude, the distance north or south of the equator from the ratio of the longest to the shortest day at that place. For longitude, he proposed a zero longitude through the island of Rhodes. The present zero longitude (called Prime Meridian) passes through the Royal

Observatory in Greenwich near London. It was established by an International Meridian Conference in 1884.

Solstices, Tropics and Festivals: Due to the Earth's tilt (of 66°) the amount of sunlight reaching any place on Earth during its orbit around the Sun varies during the year. When the Northern Hemisphere tilts towards the Sun and is closer to it, the sunlight is intense—it is Summer; and when the Earth is on the opposite side of its orbit around the Sun the Northern Hemisphere tilts away from the Sun, the sunlight is less and it is winter.

Summer Solstice (*sol*—Sun; *sisto*—I stand still) is the mid-point of summer—the day (June 22) when the Northern Hemisphere is closest to the Sun—the day is longest, and it appears the Sun stands still.

Winter Solstice is the mid-point of winter—the day (December 22) when the Northern Hemisphere is furthest from the Sun—and the night is longest. At the mid-points between Summer and Winter (March 22 and September 22)—called Equinoxes (*aequus*—equal; *nox*—nights)—the nights and days are of equal length.

Tropic from the old Greek word *tropos* meaning Turn. On June 22 the Sun is directly overhead 23°N latitude (its rays fall at ∟90°) called Tropic of Cancer, and the size of shadows is smallest. From this latitude the Sun TURNS back and its rays fall on Tropic of Cancer (and at any point in the Northern Hemisphere) at acuter angles (< ∟90°) making longer shadows. On December 22 the Sun is farthest from Tropic of Cancer and the length of the shadows is longest; while it is directly overhead Tropic of Capricon 23°S. From this day the Sun TURNS back from being directly overhead Tropic of Capricon; and after 6 months it is again directly overhead Tropic of Cancer.

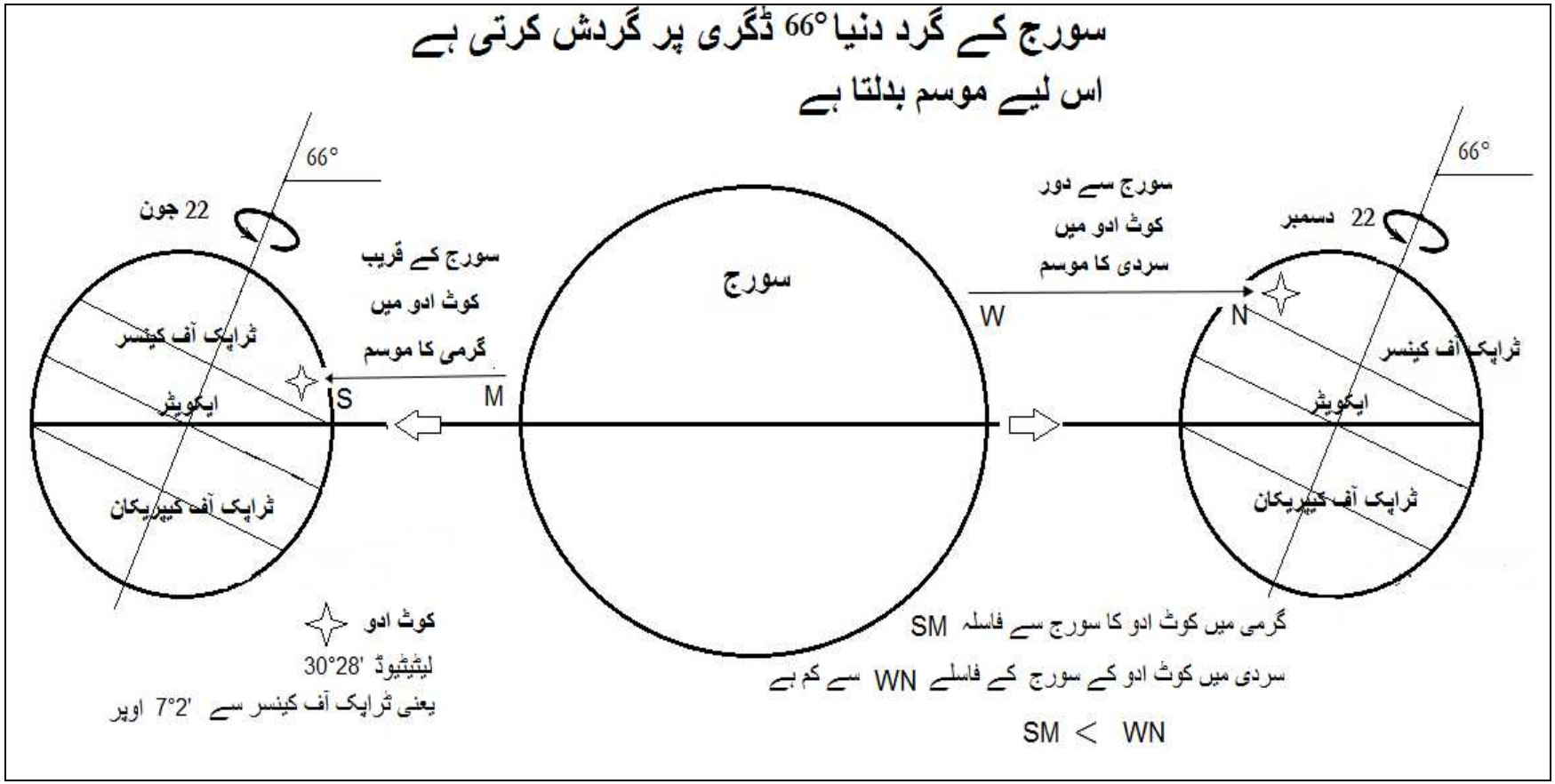
Festivals: *Nauruz* (*nav*—new; *ruz*—day) (March 21) on Spring Equinox is first day of Iran's calendar. Celebrated for over 3000 years. Houses are cleaned, new dresses are worn, and food served *haft sin* (haft—seven; sin—letter س, i.e. seven food items with names starting with the letter sin, e.g. *saeb*—apple, *sabzeh*).

Lohri: (*loh*—light and warmness of fire) celebrates the passing of winter and winter Solstice in Punjab on the shortest day of the year, which is the last day of *poh* (one of six months of Punjabi calendar). A key ancient feature of *Lohri* is the bonfire, which is common in winter solstice festivals the world over signifying the return of longer days. *Lohri* coincides with the *rabi* harvest crop (sugarcane); and marks the start of the new financial year for farmers: tenancies start on *Lohri* and rents are collected. The memory of Dulla Bhatti is recalled as

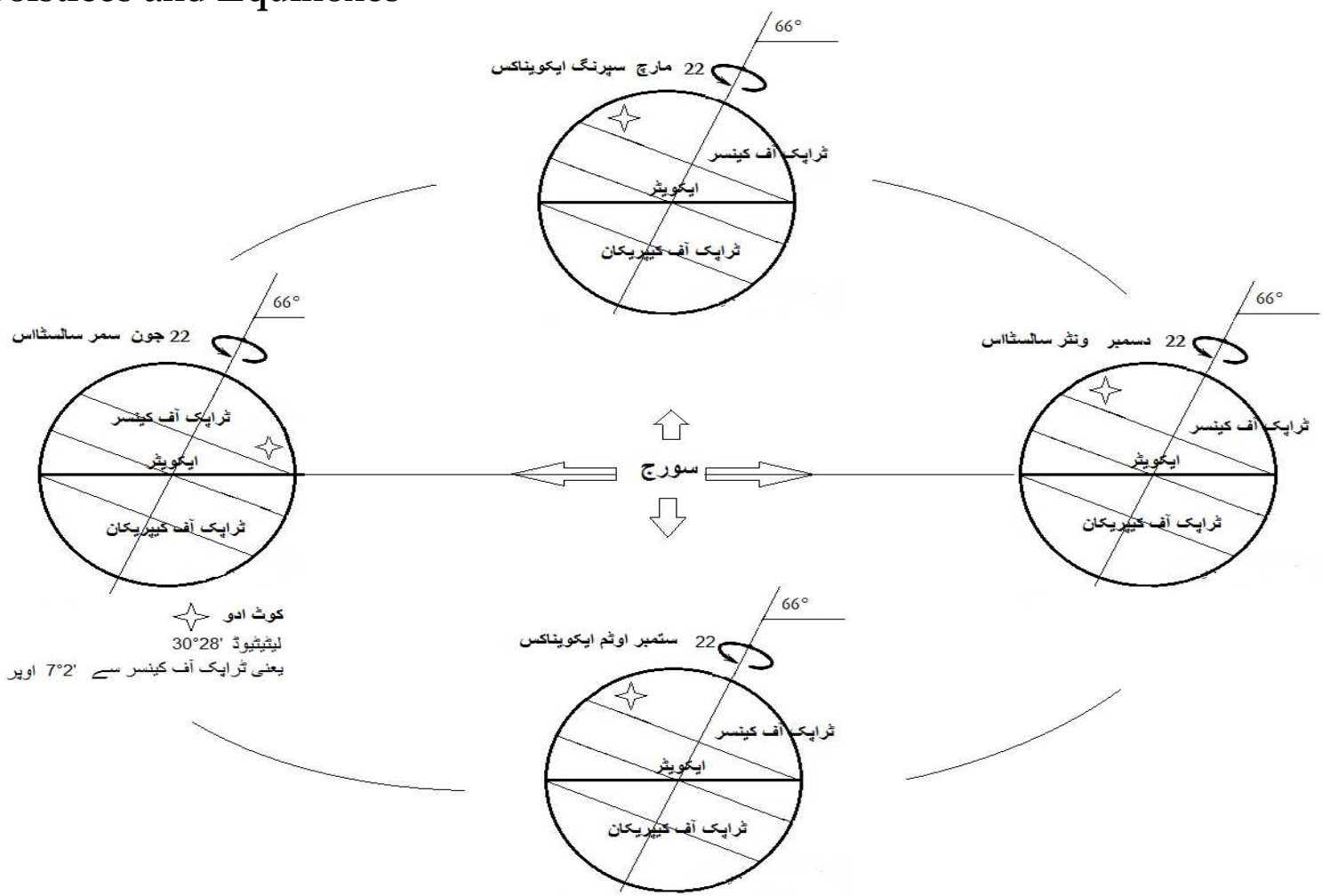
"an honorable son of Punjab who refused to sell the soil of Punjab" by reciting *vars* (old poetry put to music) praising his rebellion against Akbar, and his opposition to the abduction and selling of local girls, which was common in Mughal time.

Vaisakhi: Punjabi harvest festival at the Spring Equinox. It marks the start of the Punjabi calendar. *Bhangra* is a harvest dance.

Axial Tilt and Seasons

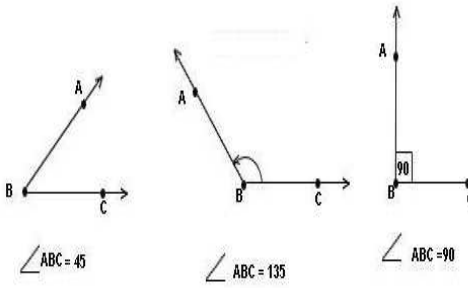


Solstices and Equinoxes



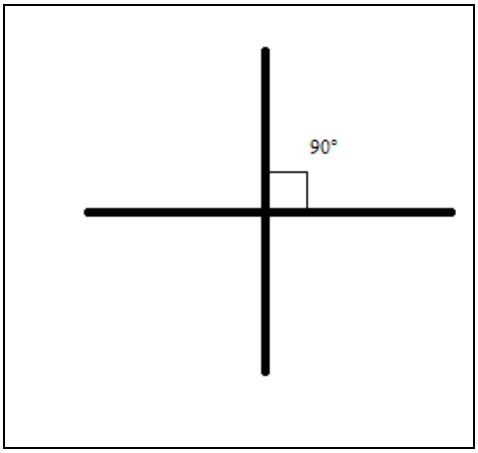
Background Poems in Seraiki

زاویہ



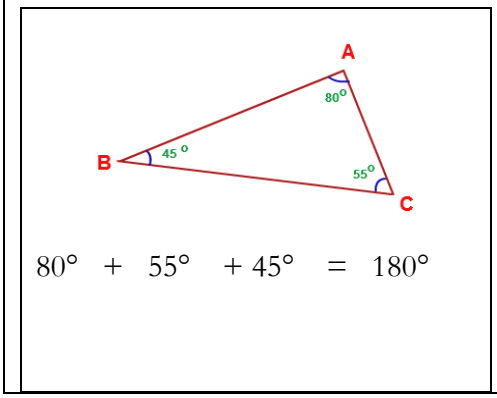
آغاز تاں ہکی جاہ ہوندے
خط خط دے نال جڑیا ہوندے
ڈو خط دا زاویہ ہوندا ہے
جیندا اپنا اپنا راہ ہوندے

عمودی خطوط



جڈاں خط عمود بنڑیندے ہن
ڈو زایے قائم کریندے ہن
او خط عمودی خط ہوندن
جیندے زاویے نوے تھیندے ہن

تکونا (مثلث)

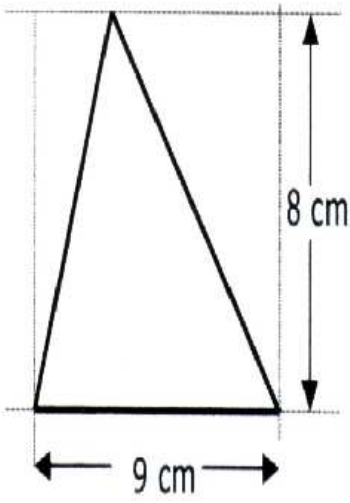


80° + 55° + 45° = 180°

ئے خط ہوون کہیں شکل وچ خط خط دے نال ملائے ہوون
ئے کونی شکل بنڑدی ہے اوندے زاویے جیکر ترائے ہوون
مع جیکر زاویں کوں کل ہک سو اسی تھئے ہوون
زاویہ تاں معلوم تھیندے معلوم جے زاویے بئے ہوون

مثلث دا رقبہ

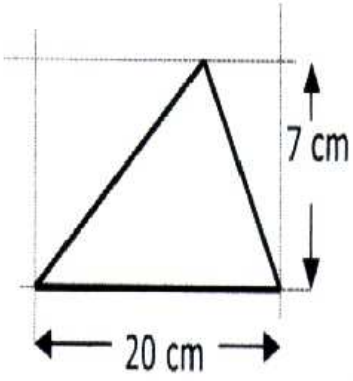
ترائے کونی کہیں وی شکل دا جڈاں رقبہ آن گولیندا ہے
کر ضرب عمود تے قاعدے کوں مجموعہ آن کڈھیندا ہے
ایں عمل دے بعد جواب اُتے ول ڈو تقسیم کریندا ہے
ترائے کونی کہیں وی شکل دا اے ٹوٹل رقبہ تھیندا ہے



رقبہ $= \frac{9 \times 8}{2}$

$= \frac{72}{2}$

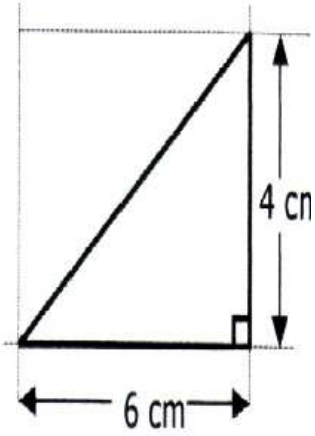
$= 36\ cm^2$



رقبہ $= \frac{20 \times 7}{2}$

$= \frac{140}{2}$

$= 70\ cm^2$

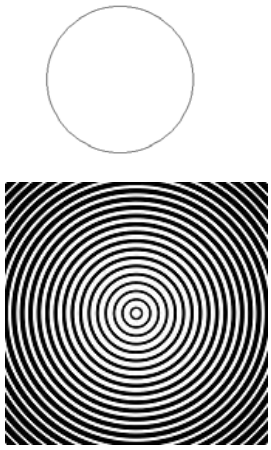


رقبہ $= \frac{6 \times 4}{2}$

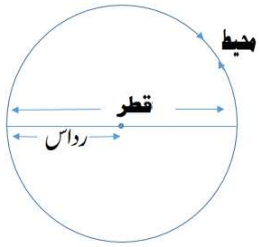
$= \frac{24}{2}$

$= 12\ cm^2$

دائرہ



اُتھ شکل دائرہ بنڑدی ہے جتھ نقطیں دے کجھ کُھ ہوندن
ہک ہوندے مرکز نقطیں دا بئے آپ محیط دی وٹھ ہوندن
ہوندا قطر دا دائرہ ہکو ہے بھانویں چھی ہوندن بھانویں اٹھ ہوندن
توڑے دائرہ چھوٹا وڈا ہے اوندے زاویے ترائے سو سٹھ ہوندن



جڈاں ناپنا ہووے دائرہ کوں قانون اے یاد رکھیندے ہن
محیط تے قطر پائی ہوندن جڈاں اے تقسیم کریندے ہن
ہک قطر برابر تاں تھیندے ڈو ضرب رداس ڈیویندے ہن
کرو قطر تے پائی کوں ضرب جیکر محیط برابر تھیندے ہن

دائرے کا مربع

=

پائی آر سکیئر

رقبہ کہیں وی دائرے دا قانون دے نال گولیندا ہے
معلوم تاں رقبہ تھی پوندے جڈاں اے قانون لویندا ہے
کر ضرب محیط رداس کوں ول ڈو تے تقسیم کریندا ہے
رکھ یاد توں رقبہ دائرے دا پائی آر سکیئر تھیندا ہے

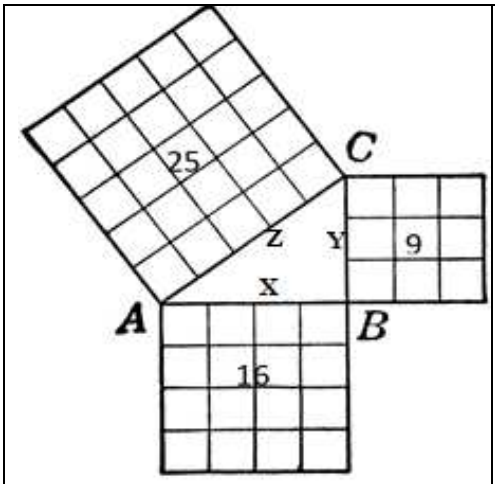
مسئلہ فیثاغورث

ترائے کونی کہیں وی شکل وچ ہک زاویہ قائم کریندا ہے

اوزاویہ نوے ڈگری تے ہر صورت ٹھیک رکھیندا ہے

کرو قاعدہ جمع عمودکوں ول مربع نال گھنیندا ہے

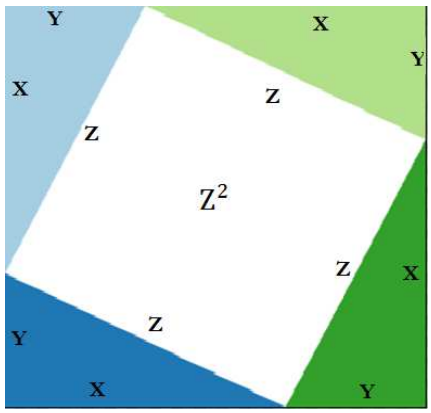
قانون ہے فیثاغورث داا ے وتر برابر تھیندا ہے

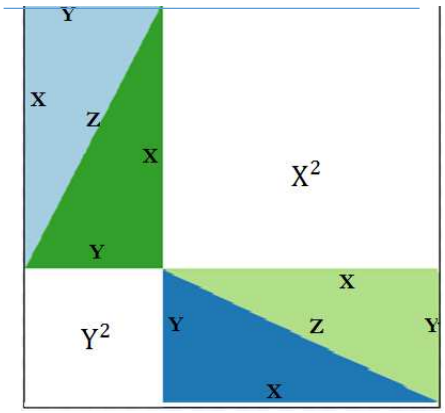


$$16 + 9 = 25$$

$$AB^2 + BC^2 = AC^2$$

$$X^2 + Y^2 = Z^2$$

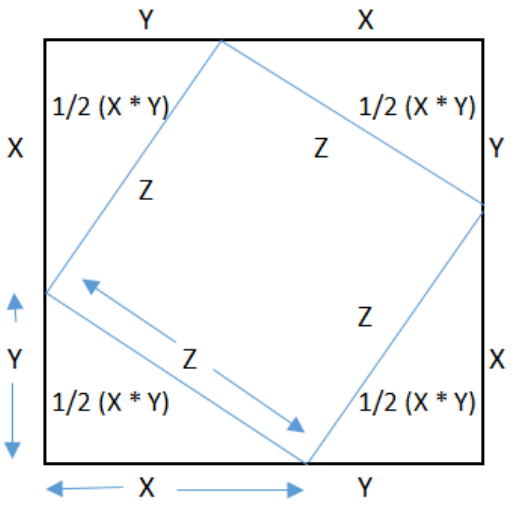




فیثاغورث دا ثبوت

حل مسئلہ فیثاغورث دا
ہووے کرنا ثابت مسئلے کوں
ایں مسئلے وچ الجبرے دا
کرو جمع حصے رقبہ دے
الجبرے وچ ایں مسئلے کوں
بیا X تے Y دے فاصلے کوں
ایندیاں چار مثلث بنڑ ویندن
X تے Y دا مربع وی

قانون دے نال گولیندا ہے
الجبرے نال کریندا ہے
بس اے قانون لویندا ہے
ہر وار مربع تھیندا ہے
ترتیب دے نال لکھیندا ہے
ہر صورت ٹھیک رکھیندا ہے
جڈاں وچ چوکور بنڑیندا ہے
پچھیں Z سکیئر تھیندا ہے



مربع دا رقبہ:

$$(X+Y)^2 = X^2 + Y^2 + 2*X*Y$$

کرو جمع حصے رقبے دے

$$= 4*(\frac{1}{2}*X*Y) + Z^2$$

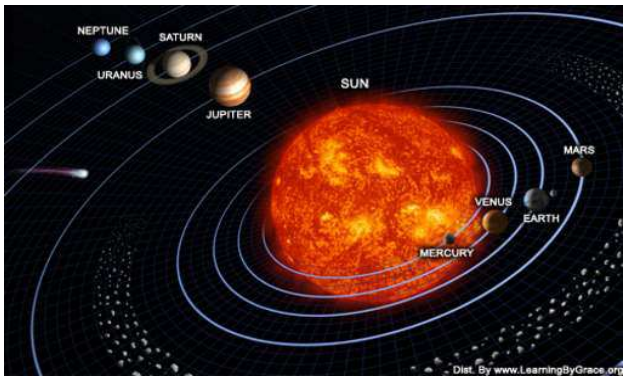
$$= 2*X*Y + Z^2$$

ہر وار مربع تھیندا ہے

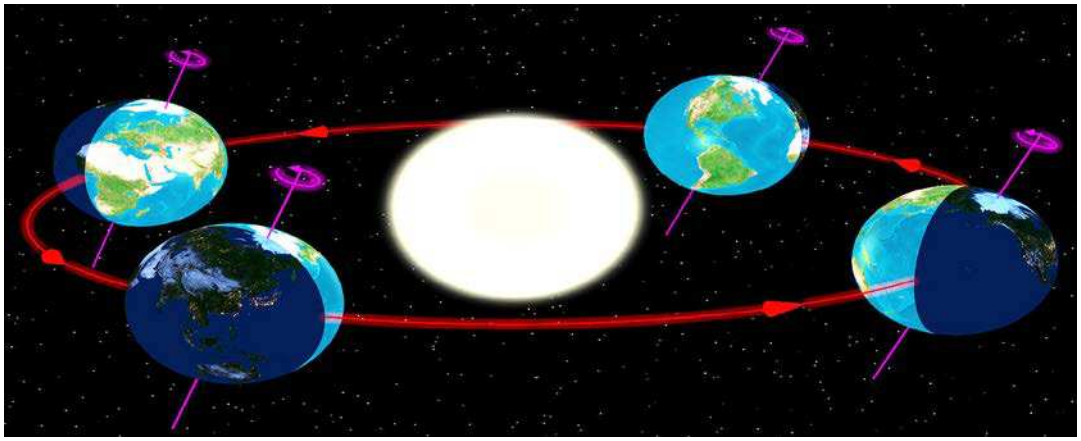
$$X^2 + Y^2 + \cancel{2*X*Y} = \cancel{2*X*Y} + Z^2$$

$$X^2 + Y^2 = Z^2$$

گلیلیو اور سائنس



کرو تحقیق ہر شے دی
ساکوں سائنس اے سکھیندی ہے
گلیلیو نے آ کھیا ہا
زمیں گردش کریندی ہے
اے سجھ مرکز ہے دنیا دا



ساکوں سائنس اے ڈسیندی ہے
ہیلیو سینٹرک دا مطلب ہے
زمیں چکر چلیندی ہے
ہے سجھ دے نال کہیں ویلے
ولا کجھ دور ویندی ہے
ونج کے دور موسم کوں

اے کر تبدیل ڈیندی ہے
زمیں 66 ڈگری تے
پئی چکر کٹیندی ہے
زمیں جو خود مدار اچ ہے
تاں سجھ مرکز بنڑیندی ا ہے
زمیں گردش کریندی ہے