

## EARTH'S CIRCUMFERENCE

**Purpose:** *Learn to use circles and triangles for global measurements. Learn about Eratosthenes, Al-Biruni, Al-Mamun, Aryabhata, Al-Idrisi and their times. See how geometry/knowledge progressed along with rational thinking/cultural exchange; And how celestial events influence culture.*

### Topics Covered:

1. Geometry/ Trigonometry: Circumference, Angles in a Circle, Days in a Year, Parallel lines and Angles, Right Angled Triangles. Trigonometric functions: Sin, Cosine, Tan
2. Geography: Latitude, Longitude, Tropics, Axial Tilt, Solstices, Equinoxes and Local Festivals associated with them: *Nauroz, Lohri, Vaisakhi*; World Map of Al-Idrisi
3. History: Eratosthenes of Alexandria, Library of Alexandria, Greek Rationalism, Arabs and Europe, Alexander, Selucus and Ptolemy, Ambhi and Raja Poru. Golden Age of India: Art and Sciences in the Gupta Empire, *Shakuntala, Panchtantra*. Al-Mamun and *Bayt Al-Hikmah*.

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Images from Wikipedia

## Earth's Circumference—Method of Eratosthenes

**Geometry** (*geo*—earth; *metry*—measurement): Eratosthenes (276—194 BC) first calculated the Earth's circumference in two steps using properties of angles and circles and observing the Sun simultaneously at two places: at Aswan (which lies on the Tropic of Cancer and so on June 22 the Sun makes a  $90^\circ$  angle with it shining directly on the bottom of a well in Aswan) and at Alexandria.

Step 1: While shining directly on Aswan (at  $90^\circ$ ) the Sun shines at an angle on Alexandria (located north of Aswan). Eratosthenes measured this angle on June 22 (at noon) by finding  $\text{Tan}^{-1}$  of the ratio of the lengths of a tower in Alexandria and its shadow, which is the same as the angle of Aswan and Alexandria from the Earth's center ( $\angle \text{COD} = 7^\circ 12'$  or  $1/50$  of  $360^\circ$ )<sup>1</sup>.

Step 2: He measured the distance between Aswan and Alexandria (800 km) (from the number of steps a camel took to go between the two cities multiplied by the length of the camel's step); And calculated the Earth's Circumference from the equality of the ratios:

$$\rightarrow \frac{800}{\text{Earth's Circumference}} = \frac{7^\circ 12'}{360^\circ} = \frac{1}{50}$$

$$\text{Earth's circumference is } 50 \times 800 = 40,000 \text{ km}$$

### Finding Angle $\angle \text{COD}$

Sun's rays AB and OD are parallel.

$$\therefore \angle \text{COD} = \angle \text{ABC}$$

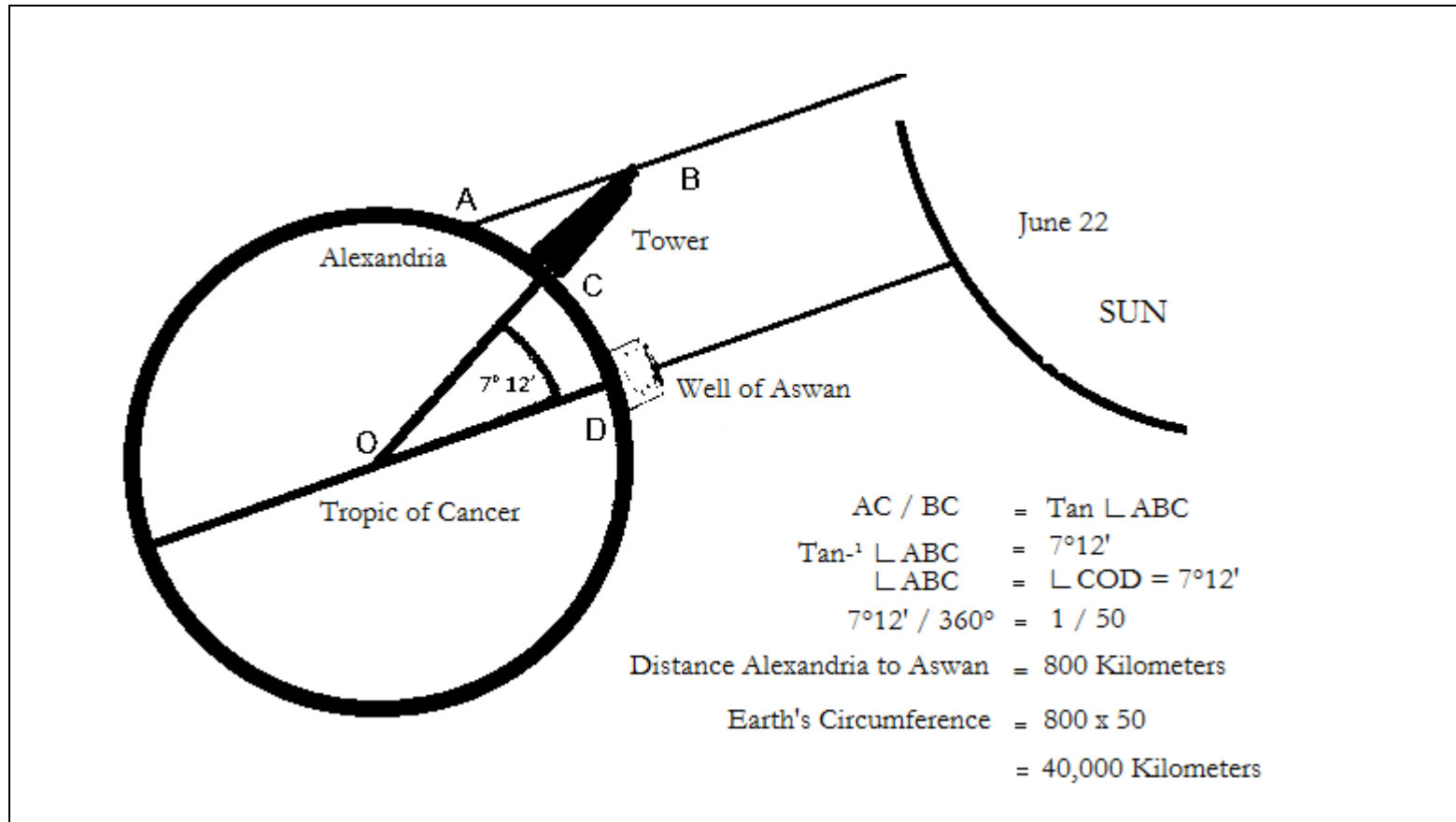
In  $\Delta \text{ABC}$  made by the sun's ray AB, the tower BC and the tower's shadow at noon AC the ratio of the perpendicular to the base

$$= \frac{\text{AC}}{\text{BC}} = \text{Tan } \angle \text{ABC}$$

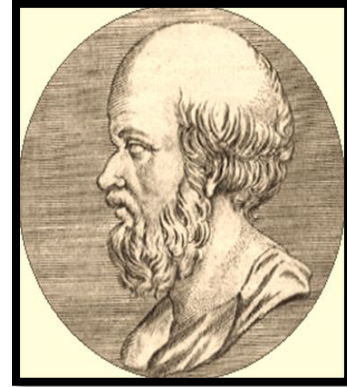
$$\therefore \angle \text{ABC} = \text{Tan}^{-1} \angle \text{ABC} = 7^\circ 12' = \angle \text{COD}$$

<sup>1</sup> Adding  $\angle \text{COD}$  to the latitude of the tropic of Cancer gives the latitude of Alexandria. See page 9 for similar calculation of latitudes of Kath by Al-Beruni and of Kot Addu.

## Eratosthenes Method to Measure Earth's Circumference



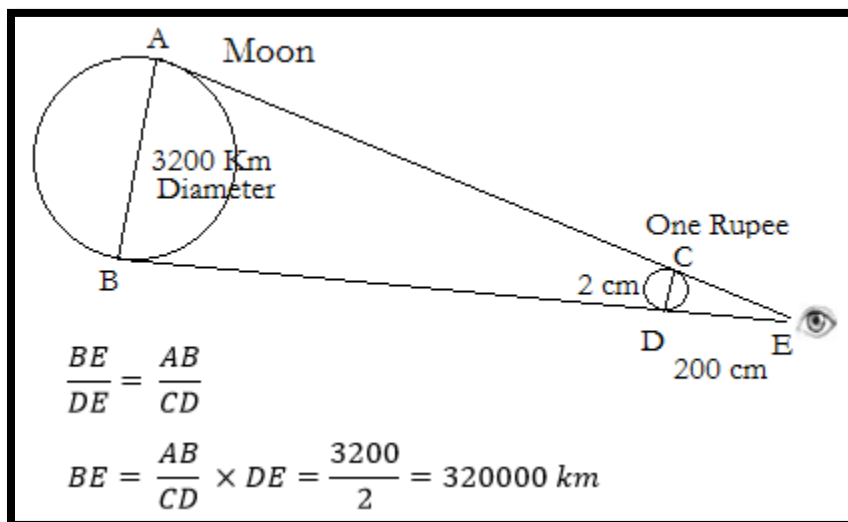
**Eratosthenes** (276—194 BC) was born in Libya. He headed the **Library of Alexandria**—the highest official appointed by the king—and tutored the king's son. The Library was part of the *Musaeum* (place of the Muses—dedicated to nine Greek goddesses or muses, daughters of Zeus, personifying arts, literature and music) where over 1000 scholars researched, lectured, collected and translated documents from India, Greece and Iran. They included Euclid—founder of Geometry (around 450—350 BC), Archimedes (287—212 BC)—founder of engineering, Aristarchus (310—230 BC), who first proposed the heliocentric system of the universe. The *Musaeum* was built by Ptolemy, Alexander's general and after him Egypt's ruler, though it was Alexander's idea to collect books from his empire into one place.



*Eratosthenes*

Eratosthenes was also a geographer. His book *Geographica* used reports of Alexander's surveyors, explorers and voyagers and included China but not North Europe, North Asia, South Africa.

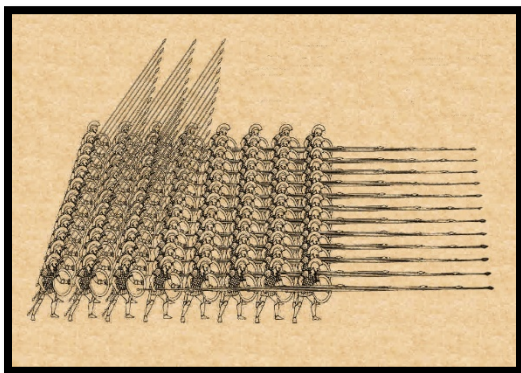
Using the estimate of Earth's circumference (and diameter) the moon's diameter is calculated—by observing, during a lunar eclipse, that the moon



takes a fourth of time to eclipse as compared with the time it takes to come out from the earth's shadow. Its diameter is thus a fourth of the earth's or 3200 km ( $= \frac{40000}{4\pi}$ ). Also, the distance to

the moon (BE in the above diagram) is calculated using a one Rupee coin (diameter 2 cm which hides a full moon when placed 200 cm from the eye) and equality of ratios of the sides of triangles ABE and CDE. The distance of the moon from earth is thus 100 times more than the moon's diameter or 320000 km.

**Greece and India:** Until Alexander's (356—323 BC) invasion of India in 326 BC Iranians ruled the territory west of Indus and perpetually fought the Greeks. Xerxes' (486—465 BC) army had soldiers from his vast empire: Indians wearing cotton dresses, Afghans, Libyans, Arabians, Egyptians, Turks, South Europeans. He subdued Greece and raided Athens in 480 BC. But 130



*Greek Phalanx*



*Greek coin showing an Indian elephant on the run before a Greek horseman*

years later a similar immense but disorderly army under Darius was beaten by disciplined Greek phalanxes supported by swift dashes of Alexander's cavalry.

Alexander built new cities in Kandahar (Afghanistan) and Khojant (Tajikistan) and here married Roxana (who bore him a child after his death). Raja Ambhi of Taxila ruled the Indus—Jhelum *doab* (area between 2 rivers) and joined him to



*Battle on the Jhelum*

advanced the cavalry encircled the battlefield. The Indian infantry crushed in the center confusing their elephants, who trampled on Indian soldiers.

defeat Raja Poru (326 BC) who ruled the Jhelum—Ravi doab. Alexander made Poru his satrap like so many kings he defeated.

In the battle on the east bank of river Jhelum Poru's cavalry was harassed by 1000 mounted archers, then attacked and decimated by Alexander's battle-hardened cavalry. As the Greek phalanxes

Alexander's army returned home via Indus and Arabian Sea and through Baluchistan. Ten thousand died crossing the Baluch desert. Alexander died in Babylon (Iraq)—the greatest conqueror of the World until Chengiz Khan.

Alexander's empire split into three after his death. Most of Asia went to his general Selucus who gave to Chandragupta Maurya the west bank of Indus upto Kandahar in exchange for peace and 500 elephants. In his book *Indica*, Megasthenes—Selucus' emissary at Chandragupta's court in Pataliputra (now Patna, 1000 km east of Delhi and 300 km south of Khatmandu) estimated the North-South length of India at 4000 km or about a third more than the actual length of 2900 km, which a generation later Eratosthenes adjusted downward by 500 km.

In Afghanistan/KP the Greeks ruled another 300 years followed by the Kushan Empire of central Asians under whose greatest king Kanishka Peshawar and Taxila became centers of learning and Buddhism. Meanwhile, in central and Northern India Chandragupta Maurya erased all trace of Alexander's authority and started the Mauryan dynasty, which ruled India and Afghanistan for 137 years, including Chandragupta's grandson Asoka. Ashoka's empire crumbled after him, but as Will Durant says he “accomplished one of the greatest tasks in history. Within two hundred years after his death Buddhism had spread throughout India.”

The Golden Age of India came in the Gupta Empire (320—550).



*Socrates was sentenced to death for corrupting the minds of the youth of Athens. In the picture he drinks the poison while his students mourn.*

### **Greek Rationalism and Islam:**

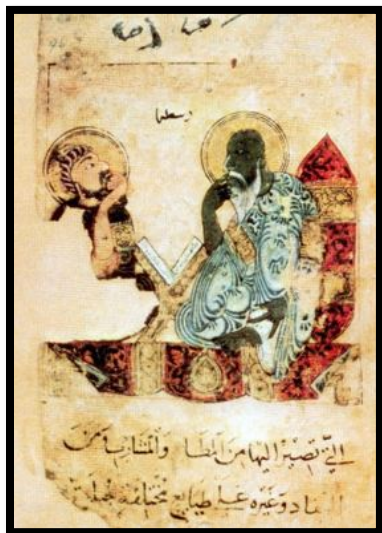
Alexander and Ptolemy studied with Aristotle (384—322 BC) a student of Plato (427—347 BC) who was a student of Socrates (470—399 BC). The Greeks used logic and reason to address matters of belief and practice—a way of thinking called *Rationalism*.

The Socratic Method (or scientific method or dialectics or logic or rational method) questions ideas and rejects them if they are found under questioning to self-contradict,

and replaced with other ideas to be similarly questioned. Science requires such thinking and observation, patient collection of facts, experimentation and boldness in framing ideas, an independent and inquisitive spirit, an ability to see things as they are.

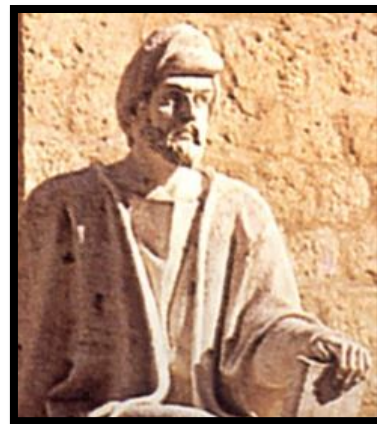


Muslim scholars called Aristotle *Al-Muallim-Al-Awwal* (First Teacher). Al-Farabi (872—950 AD), Ibn-Sina (980—1037 AD) and Al-Biruni (973—1048 AD) debated Aristotle. Influenced by Aristotle caliph Al-Mamun (786—833) and the Mutazili sought to reconcile the justice of an all-powerful God with evil in the world. The Ulema and later Ibn-Hanbal opposed them saying: *everything a believer needs to know about faith and practice is in the Quran and Hadith*. Ibn-Hanbal suffered from the Abbasids' *mihna*—punishment of Ulema opposing the Mutazili understanding of the created nature of Quran. Ibn-Hanbal rejected religious rulings (*Ijtihad*) of jurists and *bida* (innovation) as speculative theology (*Kalam*).



*Aristotle: Al-Muallim-Al-Awwal in Arab painting*

Later Ibn-Gazali (1058—1111) gave a further blow to Muslim rationalism with his book *Tahafut al-Falasifa* (*Incoherence of the Philosophers*), which denounced Al-Farabi and Ibn-Sina. Ibn Rushd (1126—1198) (called Averroes in the west) wrote *Tahafat al-Tahafat* (*Incoherence of the Incoherence*) to defend using Aristotle in Islamic thought. According to him: natural phenomenon follow laws that God created. But al-Ghazali claimed that whatever happens is because God wills it.



*Ibn-Rushd: physician, mathematician, philosopher popularizer of Aristotle*

Ibn Rushd's book was not well-received by Muslim scholars. Muslims fell into the stranglehold of Mullahs and remain in their fold. Argument from authority, indifference to facts, blind faith darken the Muslim mind. The driving force of the scientific technological revolution is creativity—the essence of *Ijtihad*.

The West became Averroists—followers of Ibn-Rushd—adopting him as the founding father of rational thought.

The Hanbali school of Islamic law is dominant in Saudi Arabia and Qatar. The Salafi movement is inspired by Hanbali sharia and strictly follows *al-salaf al-salih* (pious forefathers).

## Earth's Circumference: Calculated by Students of Zoya Science School Nala Khudadad

First: Using Eratosthenes' method the students found the angle of Kot Addu and Tropic of Cancer from the Earth's center ( $\angle COD = 7^\circ$ ) on June 22 when the sun was at  $90^\circ$  above Tropic of Cancer. This, by finding the angle made by the Sun and the shadow of a stick (and cross-checked the result by subtracting the Tropic of Cancer's latitude from Kot Addu's latitude ( $30^\circ 28' - 23^\circ 26' = 7^\circ 2'$ )).

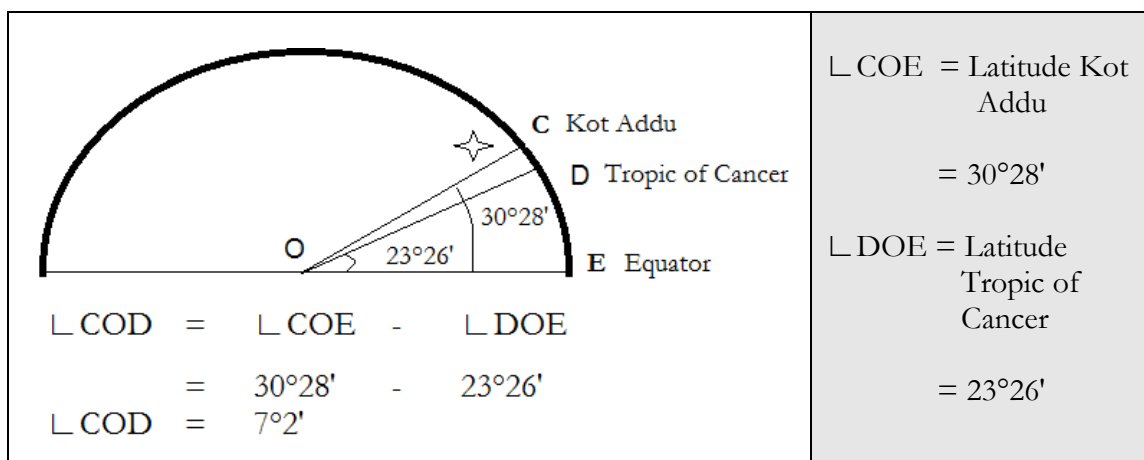
And, following Al-Mamun (page 9) they found that  $1^\circ\text{N}$  latitude covers (approximately) 107.28 km: distance between Mianwali ( $32^\circ 59' \text{ N } 71^\circ 54' \text{ E}$ ) and Kot Addu ( $30^\circ 28' \text{ N } 70^\circ 57' \text{ E}$ ) = 270 km; Difference in latitude is ( $32^\circ 59' - 30^\circ 28'$ )  $\rightarrow 1^\circ\text{N} = 270 \text{ km} \div (32^\circ 59' - 30^\circ 28')$ .

$\therefore$  distance between Kot Addu and Tropic of Cancer is 754.6 km  $[(32^\circ 59' - 23^\circ 26') \times 107.28]$ .

From the equality of ratios of an arc of a circle to its circumference and the angle of the arc to degrees in a circle:

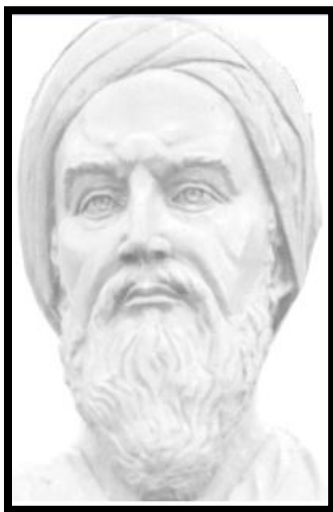
$$\rightarrow \frac{754.6}{\text{Earth's Circumference}} = \frac{7^\circ}{360^\circ} = \frac{1}{51.4}$$

Earth's circumference is  $51.4 \times 780 = 38,806 \text{ km}$





## Earth's Circumference: Other Measurements



*Abu Rayhan Al-Biruni*

**Abu Rayhan Al-Biruni** (973-1048). Al-Biruni means “from outer place” i.e. from Khwarezm (presently Uzbekistan) away from Iran. Al-Biruni calculated the Earth’s circumference from a single location using properties of triangles. First, he calculated the height (h—in the diagrams below) of a hill near Pind Dadan Khan (from the angles  $\angle A$  and  $\angle B$  the hill top makes with two points on the plain next to the hill, and the distance ‘d’ between the two points); Then he measured the angle ( $\angle a$ ) the hill top makes with the horizon. And using trigonometric functions he expressed the radius (and circumference) of the Earth in terms of ‘h’ and ‘ $\angle a$ ’. Al-Biruni estimated the Earth’s radius to be 6340 km (and circumference 39838 km). A similarly accurate estimate was calculated in Europe only 500 years later.

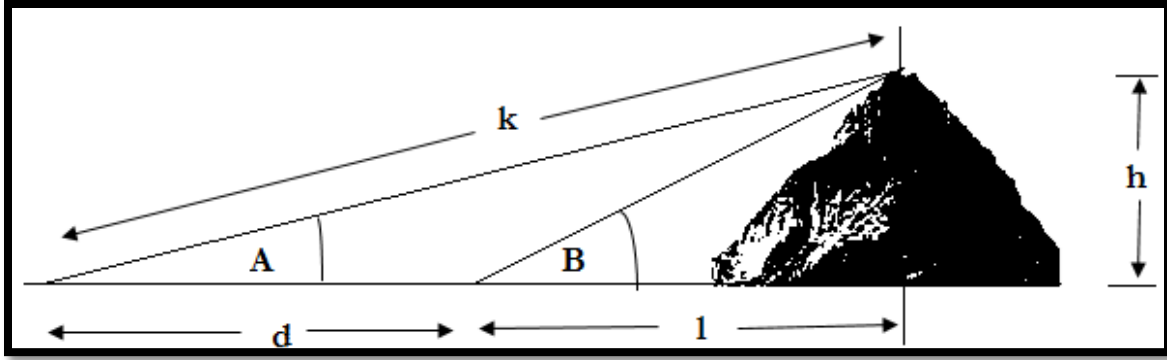
At age 17 Biruni calculated the latitude of Kath (capital of Khwarazm) using the maximum altitude of the Sun. Similarly, the latitude of Kot Addu =  $30^\circ$ —the angle made by the Sun on Kot Addu on June 22 i.e.  $7^\circ$  plus  $23^\circ$  (the latitude of Tropic of Cancer where the Sun shines at  $90^\circ$  on this day).

His book (one of 103) *Kanoon Masudi* (encyclopedia of geography and astronomy) gives the coordinates (latitudes and longitudes) of six hundred places, which he visited. In 1017 and later he travelled with Mehmud Gazni to India and wrote *Tarikh Al-Hind* in 1030 which studies Hinduism and its relationship with the Greeks. He studied religions and saw different cultures as related and interlinked because according to him: *they are made by humans who are all related*.

**How Al-Biruni calculated the height of the hill** or expressing ‘h’ in terms of  $\angle A$ ,  $\angle B$  (angles made by two points on the ground with the tip of the mountain) and ‘d’ (distance between the two points) which are easily measured.

$$\frac{h}{d + l} = \tan A$$

$$\therefore h = d \tan A + l \tan A \dots\dots\dots (1)$$



*Calculating the height of the hill from angles A and B and the distance between them*

$$\begin{aligned} \text{Also } \frac{h}{l} &= \tan B \\ \therefore l &= \frac{h}{\tan B} \end{aligned}$$

Inserting value of  $l$  in equation (1) gives

$$h = d \tan A + \frac{h}{\tan B} \tan A$$

$$h - \frac{h \tan A}{\tan B} = d \tan A = \frac{h}{\tan B} (\tan B - \tan A)$$

$$\frac{h}{\tan B} (\tan B - \tan A) = d \tan A$$

$$\therefore h = \frac{d \tan A \tan B}{\tan B - \tan A}$$

**Calculating radius 'r' of Earth and circumference** when  $\angle DAB = \angle a$  and 'h' the height of the hill are known. Also known that  $\angle DAC = 90^\circ$  and since line AB is tangent to the Earth's horizon at B  $\angle ABC = 90^\circ$

$$\sin \angle BAC = \sin (90 - a) = \cos a = \frac{r}{r + h}$$

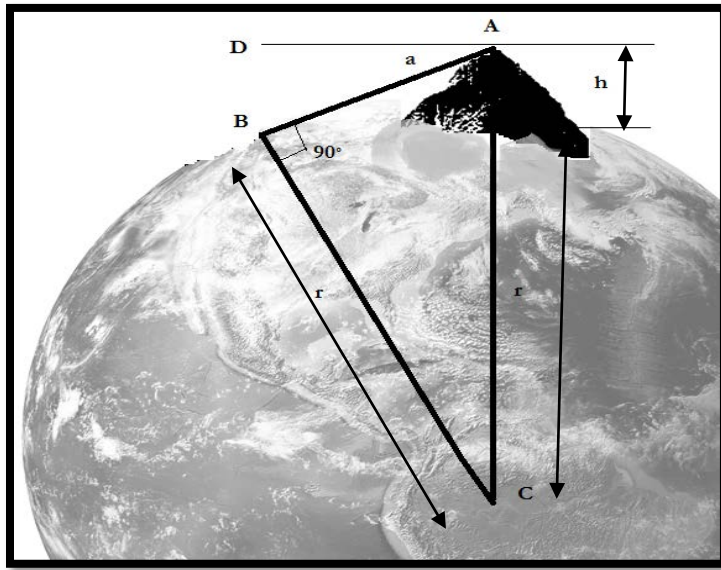
$$\therefore r \cos a + h \cos a = r$$

$$r - r \cos a = h \cos a$$

$$r(1 - \cos a) = h \cos a$$

$$\therefore r = \frac{h \cos a}{(1 - \cos a)}$$

$$\text{and Earth's circumference} = \frac{2\pi h \cos a}{(1 - \cos a)}$$



*Calculating radius of the Earth using angle  $a$  and the height of the hill*

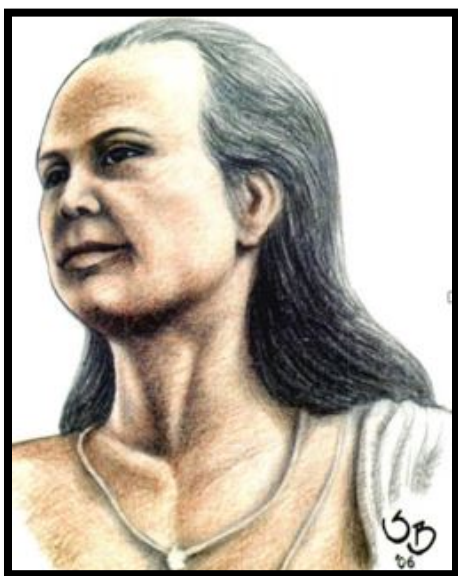
**Al-Mamun's** (786—833) team of geographers in *Bayt-al-Hikmah* (House of Knowledge) in Baghdad measured how much  $1^\circ$  latitude ( $34^\circ$  and  $35^\circ\text{N}$ ) is in miles (= 111.7 km) and multiplied it by 360 to find the Earth's circumference: For this purpose two teams set out North and South in a field near Mosul (in Iraq) until the angle of the Sun changed by  $1^\circ$  (corrected for the Sun's declination) and measured the distance covered.

Al-Biruni says in *Kitab Tahdid Al-Amakin* (*Coordinates of Cities*) that Al-Mamun's team knew from Greek books that  $1^\circ$  latitude is 500 *stadia* but it could not determine the length of the *stade* (length of sports stadium) from the translations<sup>2</sup>.

<sup>2</sup> Al-Mamun's *Bayt* was a center of science learning and library with Arab translations of Greek books on philosophy, science, mathematics. It included Ptolemy's *Al-Majisti* (the greatest) (English--*Almagest*) the encyclopedia of Greek Astronomy, Mathematics and Science with description of the motion of stars and planets, and estimates of distances between them. The *Bayt* was destroyed by the Mongols in 1258.

If similar teams set out from Kot Addu (30.4°N), the one going North on the Karor-Bhakkar Road would stop in Solhanwala (31.4°N) (road distance 108 km from Kot Addu) and the one going South on Alipur-Jatoi road would stop in Alipur (29.4°N) (road distance 128 km from Kot Addu). The Solhanwala team would calculate the Earth's circumference to be  $360 \times 108 \text{ km} = 38880 \text{ km}$ ; and the Alipur team  $360 \times 128 \text{ km} = 46080 \text{ km}$ .

Al-Mamun's scholars knew the mathematical astronomy of **Aryabhata** (476—550) as well about his estimate of the Earth's circumference of 3393 *yogana* (error of 1 percent) from his text *Aryabhatia* which summarizes Indian



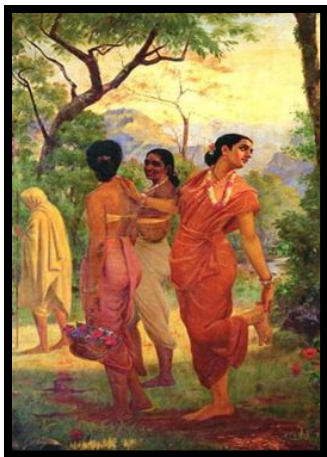
*Aryabhata. India's 1st space satellite (1975-1992) was named Aryabhata*

mathematics in 118 verses. It covers arithmetic, zero, calculation of  $\pi$ , algebra, plane and spherical trigonometry. As well, quadratic equations, sums of power series and table of Sines—preceding Al-Khwarizmi's *Al-Jabr wal Muqabila* by about 300 years. A lot of Al-Kwarizmi's Algebra came from Indian sources.

*The Earth making a revolution produces a daily rising and setting of the stars and planets. The apparent motion of heavenly bodies is only an illusion just as a passenger in a boat moving downstream sees the stationary (trees on the river banks) as traversing upstream, so does an observer on earth see the fixed stars as moving towards the west at exactly the same speed (at which the earth moves from west to east)—from Aryabhatia Section Gola 9).*

Two lyrical examples of Indian Algebra (from Will Durant): *Out of a swarm of bees one-fifth part settled on a Kadamba blossom; one third on a Silindhra flower; three times the difference of those numbers flew to the bloom of a Kutaja One bee, which remained, hovered about in the air. Tell me, charming woman, the number of bees..... Eight rubies, ten emeralds, and a hundred pearls, which are in thy ear-ring, my beloved, were purchased by me for thee at an equal amount; and the sum of the prices of the three sorts of gems was three less than half a hundred; tell me the price of each, auspicious woman.*

**Golden Age of India.** Aryabhata lived in the **Gupta empire** (320—550) during which science and arts flourished: Al-Mansur (753–774) got books on mathematical astronomy and arithmetic by Brahmagupta (598-668). Its Arabic translations by Al Fazari are called *Sindhind* and *Arakand*. From them the Arabs adopted Indian numerals and learned astronomy. According to Will Durant



*Shakuntala*

“The miscalled ‘Arabic’ numerals are found on the Rock Edicts of Ashoka (256 BC) a thousand years before their occurrence in Arabic literature.” And Simon Laplace (1749—1827) (known as the French Newton) writes: “It is India that gave us the ingenious method of expressing all numbers by ten symbols, each receiving a value of position as well as an absolute value.”

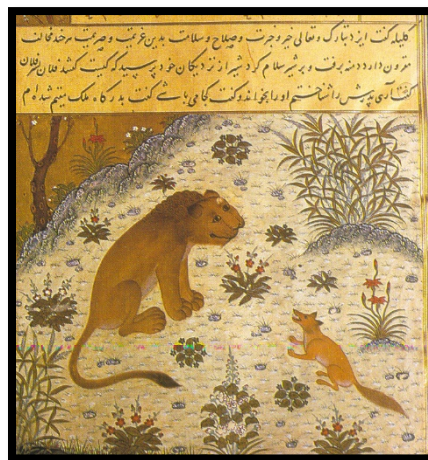
Also in India’s Golden Age Kalidas wrote *Shakuntala* (protected one) the story of a girl abandoned in a forest and protected by the birds. She is found and

raised by kind people eventually to marry a king. *Shakuntala* influenced Goethe; and turned into operas and music by western composers (such as Franz Schubert 1797—1828 of Austria).

Vishnu Sharma wrote *Panchatantra* (five texts) a collection of stories—one of the most translated story books in history. Its Arabic translation by Al-Mansur was best seller in Baghdad. The story tells how Vishnu Sharma taught wisdom to three foolish sons of a king by narrating meaningful stories.

*Susruta Samhita* a classical Sanskrit text on medicine was translated in Al-Harun’s *Bayat* into Arabic as *Kitab-e-Susrud*. It had 84 chapters covering 1120 illnesses, 700 medicinal plants, and 122 preparations (its explanation of diseases, such as, hypertension, bladder stones, obesity, diabetes matches modern symptoms).

Al-Razi’s *Al-Hawi fi tib* containing 25 volumes written in 900 which has ‘much Indian knowledge’ from *Susruta Samhita* was translated into Latin in 1279 for the French King of Sicily and called *Liber Continens*.

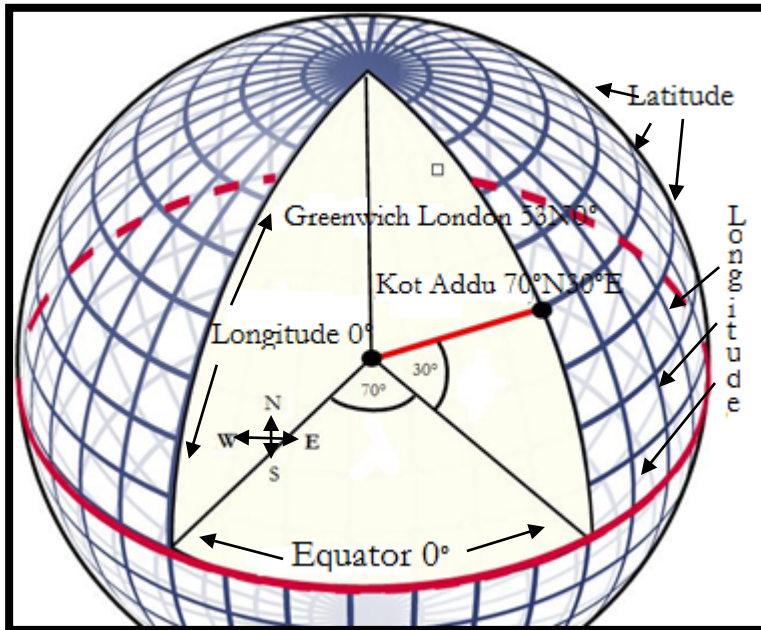


*Iranian translation of Panchatantra showing the jackal-vizir trying to lead his lion-king into war*



**Geography:** *Geo* (Earth) *graphein* (write, description) is about the processes and patterns in the earth. Eratosthenes first used the word *geographica*.

**Mapping the World:** his book *Geographica* has maps of places travelled by Greek sailors. Abdullah Ibn Al-Idrisi (1154-1192) an Arab Muslim made the 1<sup>st</sup>



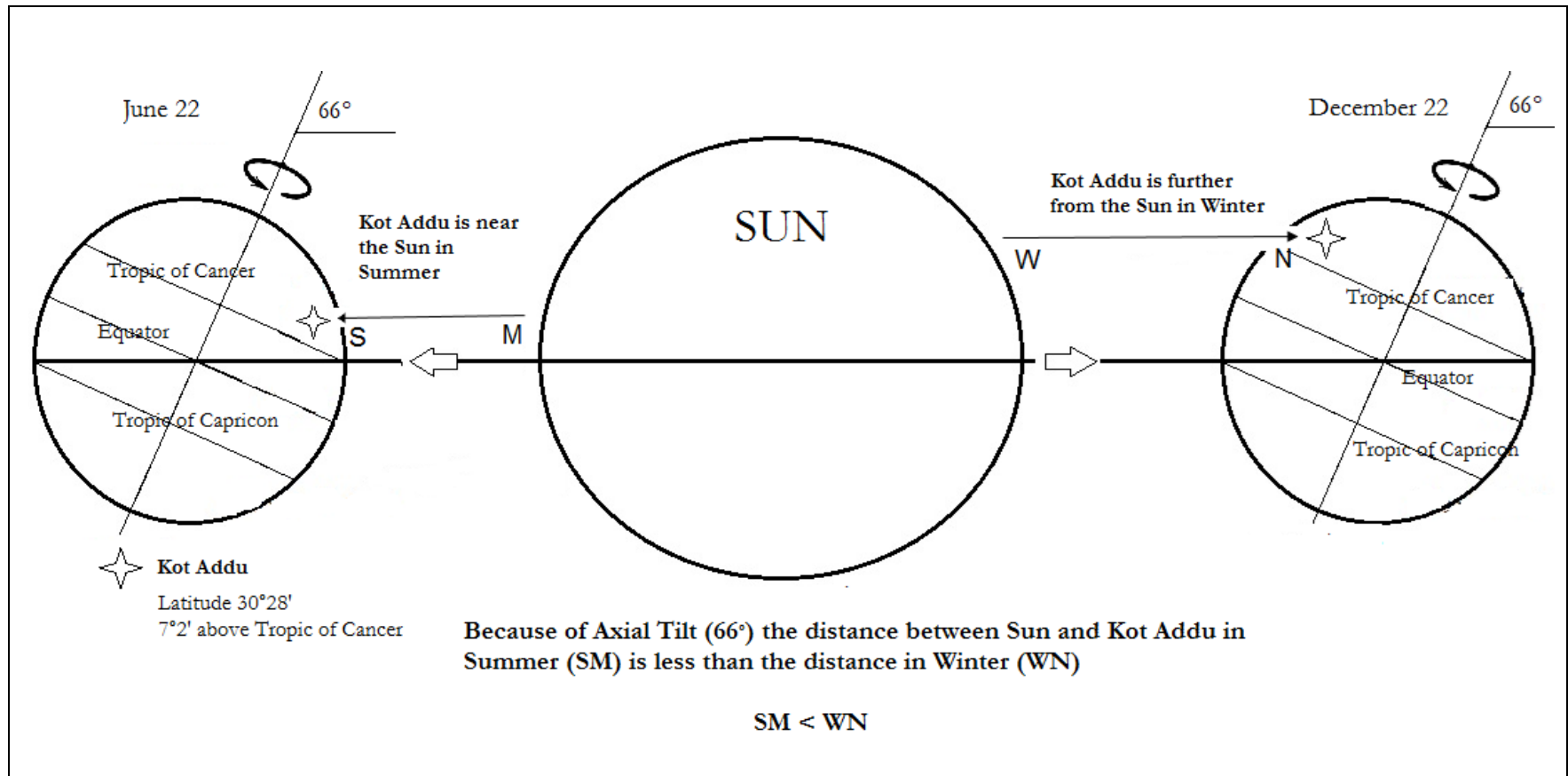
Earth's cross-section showing latitudes and longitudes and how they are determined. Kot Addu ( $30^{\circ}\text{N}70^{\circ}\text{E}$ ) is  $30^{\circ}$  North from an (imaginary) line from the Earth's center to the Equator; and  $70^{\circ}$  East of the Prime Meridian (zero longitude)

copy is in the museum of the University of Strasbourg in France) by Al-Khwarizmi who worked in Al-Mamun's *Bayat Al-Hikmah*. Al-Mamun (786—833) had also commissioned a world map with Al-Khwarizmi. It was published as an encyclopedia and is presently in *Topkapi Serai* (Istanbul museum). It has recognizable outlines of Europe, Asia and Africa as seen by Arab sailors (who by this time had reached the shores of China). **Latitude:** Al-Idrisi's map has circles parallel to the equator (latitudes) and circles going through North and South poles (longitudes) to give the positions of places on the Earth. His zero longitude goes through Mecca. Hipparchus (190—120 BC) first specified the positions of places on the Earth using coordinates to which his work on spherical trigonometry led him. He suggested measuring latitude, the distance north or south of the equator from the ratio of the longest to the shortest day at that place.

comprehensive world map on which he worked over 15 years with the Christian king Roger II of Sicily. It was published in Arabic and called *Nuzbat Al Mushtaq* and *Tabula Rogeriana* (in Latin). The map was made from first hand observations and older Greek and Iranian maps (which were accurate descriptions of closed seas, such as Mediterranean); as well *Kitab Surat Al-Ard* (Book of the Description of the Earth—a lone surviving



## Axial Tilt and Seasons



For longitude, he proposed a zero longitude through the island of Rhodes. The present zero longitude (*Prime Meridian*) goes through the Royal Observatory in Greenwich near London. It was established by an International Meridian Conference in 1884.

**Solstices, Tropics and Festivals:** Due to the Earth's tilt (of  $66^\circ$ ) the amount of sunlight reaching any place on Earth during its orbit around the Sun varies during the year. When the Northern Hemisphere tilts towards the Sun and is closer to it, the sunlight is intense—it is Summer; and when the Earth is on the opposite side of its orbit around the Sun the Northern Hemisphere tilts away from the Sun, the sunlight is less and it is winter.

*Summer Solstice* (*sol*—Sun; *sisto*—I stand still) is the mid-point of summer—the day (June 22) when the Northern Hemisphere is closest to the Sun—the day is longest, and it appears the Sun stands still.

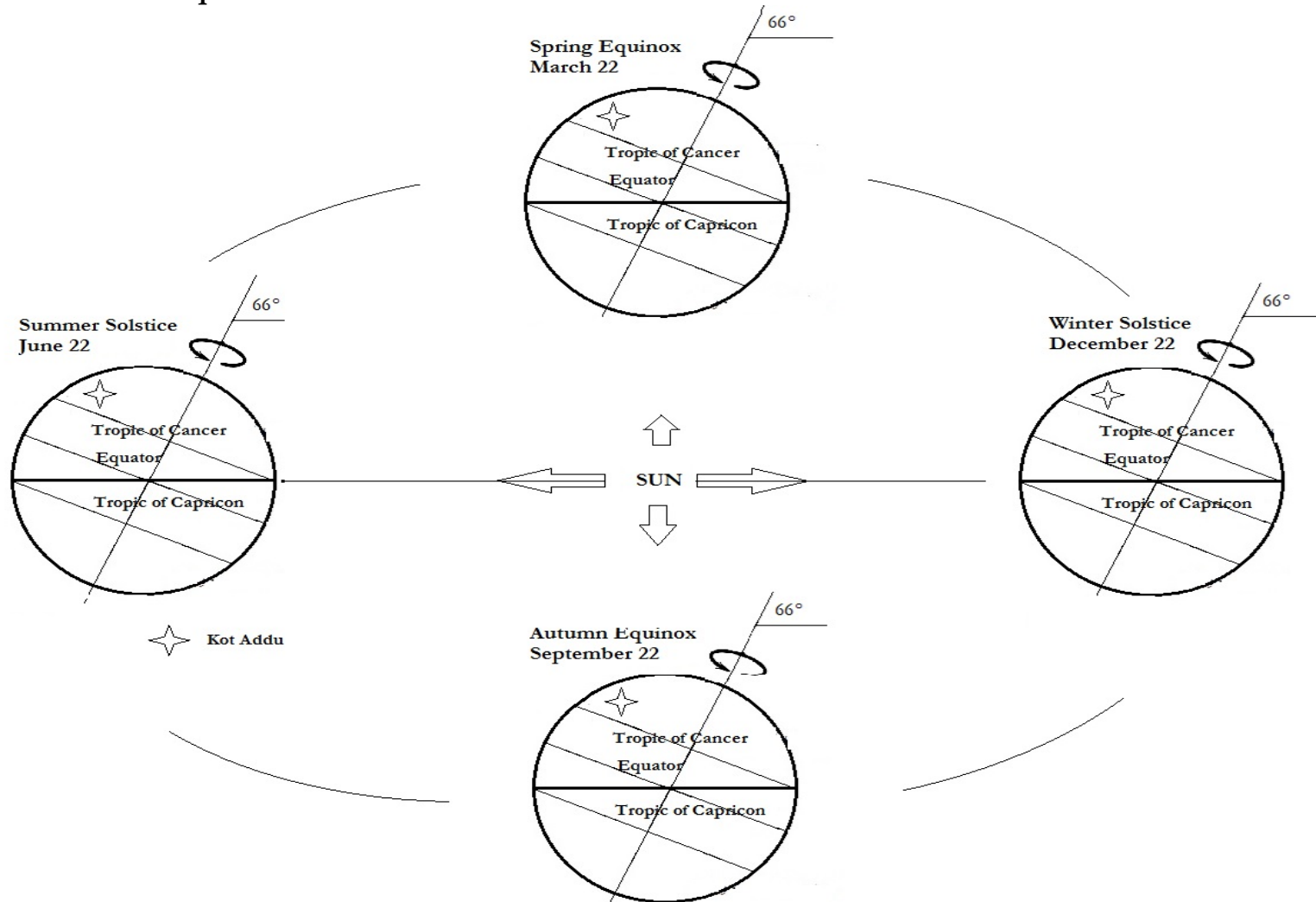
*Winter Solstice* is the mid-point of winter—the day (December 22) when the Northern Hemisphere is furthest from the Sun—and the night is longest. At the mid-points between Summer and Winter (March 22 and September 22)—called Equinoxes (*aequus*—equal; *nox*—nights)—the nights and days are of equal length.

**Tropic** from the old Greek word *tropos* meaning Turn. On June 22 the Sun is directly overhead  $23^\circ\text{N}$  latitude (its rays fall at  $\angle 90^\circ$ ) called Tropic of Cancer, and the size of shadows is smallest. From this latitude the Sun TURNS back and its rays fall on Tropic of Cancer (and at any point in the Northern Hemisphere) at acuter angles ( $< \angle 90^\circ$ ) making longer shadows. On December 22 the Sun is farthest from Tropic of Cancer and the length of the shadows is longest; while it is directly overhead Tropic of Capricorn  $23^\circ\text{S}$ . From this day the Sun TURNS back from being directly overhead Tropic of Capricorn; and after 6 months it is again directly overhead Tropic of Cancer.

**Festivals:** *Nauruz* (*nav*—new; *ruz*—day) (March 21) on Spring Equinox is first day of Iran's calendar. It has been celebrated for over 3000 years. Houses are cleaned, new dresses are worn, and food served *haft sin* (haft—seven; sin—letter س, i.e. seven food items with names starting with the letter sin, e.g. *saeb*—apple, *sabzeh*).

*Lohri*: (*loh*—light and warmth of fire) celebrates the passing of winter and winter Solstice in Punjab on the shortest day of the year, which is the last day of *poh* (one of six months of Punjabi calendar).

## Solstices and Equinoxes



A key ancient feature of *Lohri* is the bonfire, which is common in winter solstice festivals the world over signifying the return of longer days.

*Lohri* coincides with the *rabi* harvest crop (sugarcane); and marks the start of the new financial year for farmers: tenancies start on *Lohri* and rents are collected. The memory of Dulla Bhatti is recalled as “an honorable son of Punjab who refused to sell the soil of Punjab” by reciting *vars* (old poetry put to music) praising his rebellion against Akbar, and his opposition to the abduction and selling of local girls, which was common in the time of the Mughals.

*Vaisakhi*: Punjabi harvest festival at the Spring Equinox. It marks the start of the Punjabi calendar. *Bhangra* is a harvest dance.